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Estimation of modal parameters for structurally damped systems using wavelet transform

H. Sarparast [a], M.R. Ashory [a,b,*], M. Hajiazizi [c], M. Afzali [d], M.M. Khatibi [d]

[a] School of Mechanical Engineering, Semnan University, Semnan, Iran
[b] School of Mechanical Engineering, Tehran University, Tehran, Iran
[c] School of Civil Engineering, Semnan University, Semnan, Iran
[d] Department of Mechanical Engineering, Semnan Branch, Islamic Azad University, Semnan, Iran

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ABSTRACT

In this paper, a procedure is presented to determine the modal parameters of a system with proportional structural damping exited by an impact force. A Morlet wavelet transform with an adjusting parameter is used to estimate the natural frequencies and damping factors from free-decay responses of structure. Also a method is used for identification of mode shapes from the estimated natural frequencies of wavelet transform and the free responses of structure. It is shown that the mode shape identification method can average the data in the noisy environments and reduce the effect of noise. A numerical example as well as two experimental case studies on a beam and a trapezoidal concrete plate demonstrates the validity of method.

1. Introduction

Modal parameters consisting of natural frequencies, damping factors and mode shapes are known as essential data for analyzing the dynamic behavior of structures. Therefore, identification of them is a basic problem in structural dynamics. There are different theoretical and experimental approaches for determining the modal parameters. The theoretical method of Finite Element Method (FEM) is one of the well-known numerical procedures used for this task. However, FEM has deficiencies in dealing with some modeling problems such as modeling of joints. In contrast, modal testing is a practical tool for estimating the modal parameters with more precise results. Different methods have been introduced for extracting the modal parameters from experimental data with two categories of Single Degree of Freedom (SDOF) methods and Multi Degrees of Freedom (MDOF) methods (Feng et al., 1998). Pick Peaking method, Circle fit method and Line fit method are among the traditional SDOF methods (Ewins, 2000). Three points finite difference method (Yin and Duhamel, 2000) has recently been proposed with more accurate results. Within MDOF method category: Least Square Complex exponential method (Smith, 1981) poly reference time domain method (Zhang et al., 1987) Ibrahim time domain method (Ibrahim and Mikulcik, 1977) automated parameter identification and order reduction for discrete time series (Hollkamp and Batill, 1991) can be mentioned.

These methods are mostly based on Fourier Analysis with the output of Frequency Response Functions (FRFs). However, there are two important drawbacks for the methods based on Fourier transform. Firstly the modal parameters cannot be determined precisely in the noisy environments, although some approaches consisting of pre-filtering of signals can enhance the results. Secondly, the techniques with the basis of Fourier Analysis can hardly identify close modes. Wavelet Transform (WT) is a recent alternative for Fourier transform which can solve these two problems. Fourier transform has a uniform resolution in frequency domain while WT has a double resolution in both time and frequency domains (Miranda, 2008). This property of WT allows an adjustment for wavelets to analyze non-stationary signals and also identify strongly coupled modes. Moreover, WT has an inherent ability in filtering out the noise contaminating a signal (Miranda, 2008). These properties make WT attractive for determination of modal parameters. A method was proposed in Ruzzene et al. (1997) for determination of natural frequencies and damping ratios from free vibration using WT. Three approaches were presented in Staszewski (1997) for estimation of damping ratios using Continuous Wavelet Transform (CWT), Lardies and Gouttebroze (2002) has introduced a modified Morlet wavelet with a tuning parameter to improve the accuracy of the estimated natural frequencies and damping factors. A procedure for determination of modal parameters using Cauchy wavelet has been proposed by Argoul et al.

* Corresponding author. School of Mechanical Engineering, Tehran University, Tehran, Iran. Tel./fax: +98 2313354122.
E-mail address: mr.ashory@gmail.com (M.R. Ashory).

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(2000), Argoul and Le (2003a, 2003b) and Le and Argoul, (2004). A modified version of Morlet wavelet presented by Lardies and Gouttebroze (2002) has been proposed by Lardies and Ta (2011) in which the entropy of WT is minimized in order to obtain the optimal value of adjusting parameter. It has been shown that this approach improves the time-resolution of the WT (Lardies and Ta, 2011). In the above mentioned methods the mode shapes are estimated from the modulus of WT of the signals. However, the modulus of WT is vulnerable to noise (Lardies and Gouttebroze, 2002) while the estimated natural frequencies are less sensitive to noise.

In this article a procedure, based on WT, is presented for estimation of the modal parameters for the systems with proportional structural damping exited by an impact force. A Morlet transform with a tuning parameter is selected for this analysis. Also a technique is used to estimate the mode shapes from the free-decay responses of the structure and the estimated natural frequencies from wavelet. It is shown that the mode shape identification method is resistant to noise. Numerical as well as experimental case studies validate the method.

2. Theory

2.1. Time response of a structurally damped system excited by an impact force

The governing equation of the forced vibrations for a Multi Degrees Of Freedom (MDOF) system with proportional viscous damping (Ewins, 2000) is:

$$M\ddot{x} + C\dot{x} + Kx = f \tag{1}$$

where M, C and K are the mass, proportional viscous damping and stiffness matrices respectively. Also f is the excitation force vector in which one of the elements is proportional to delta function ($\delta(t)$) and the others are zero demonstrating that a point on the structure is excited by a perfect impact force while the other points are not excited. this means:

$$f(t) = \gamma \cdot \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \delta(t) \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{2}$$

where γ is a real constant. It is assumed that the system is at rest for $t < 0$(Gaul et al., 1989).

The modal coordinate vector is defined as:

$$q(t) = \phi^{-1}x(t) \tag{3}$$

where ϕ is the mass-normalized mode shape matrix.

Using Eq. (3) and the orthogonally properties of mode shapes, Eq. (1) can be decoupled as:

$$m_r\ddot{q}_r + c_r\dot{q}_r + k_r q_r = p_r, \quad r = 1,2,\dots,n \tag{4}$$

where q_r is the r th modal coordinate, m_r is the r th diagonal element of matrix $\phi^T M\phi$, which is equal to unity for mass-normalized mode shapes, c_r is the rth diagonal element of matrix $\phi^T C\phi$, k_r is the rth diagonal element of $\phi^T K\phi$ which is equal to the square of the rth undamped natural frequency, $\omega_{n_r}^2$, n is the number of degrees of freedom and p_r is defined as:

$$p_r = \sum_{i=1}^{n} \varphi_i^r f_i = \gamma\phi_l^r \delta(t) = G_r\delta(t), \quad r = 1,2,\dots,n \tag{5}$$

where p_r is the rth modal force and φ_l^r is the lth component of the rth mode shape.

The structural damping ratio is defined as (Crandall, 1970):

$$\eta_r = c_r|\omega|/k_r \tag{6}$$

where η_r is the rth structural damping ratio. Considering Eqs. (5) and (6) and the Fourier transform of Eq. (4), we have:

$$Q_r(\omega) = P_r(\omega)\Big/\left(\omega_{n_r}^2(1 + j\eta_r\omega/|\omega|) - \omega^2\right) \tag{7}$$

where ω_{n_r} is the rth undamped natural frequency and $P_r(\omega)$ is:

$$P_r(\omega) = G_r \tag{8}$$

G_r is a constant coefficient, given in Eq. (5).

In order to determine $q_r(t)$, the time response, the inverse Fourier transform of $Q_r(\omega)$ is used as:

$$q_r(t) = (G_r/2\pi)\int_{-\infty}^{+\infty} \left(e^{j\omega t}\Big/\left(\omega_{n_r}^2(1 + j\eta_r\omega/|\omega|) - \omega^2\right)\right)d\omega \tag{9}$$

where $q_r(t)$, the impulse response function, is actually the particular solution of Eq. (4) (Sun and Lu, 1995).

Eq. (9) has singularity at $\omega = 0$, therefore, the integral can be calculated using the residue theorem for which the poles of the integrand in Eq. (9) are required. These Four poles are:

$$\begin{aligned} \omega_{1,2} &= \pm(\mu_r + j\lambda_r)\omega_{n_r} \\ \omega_{3,4} &= \pm(\mu_r - j\lambda_r)\omega_{n_r} \end{aligned} \tag{10}$$

where μ_r and λ_r are defined as below (Gaul et al., 1985):

$$\mu_r = \sqrt{\left(\sqrt{1+\eta_r^2}+1\right)\Big/2}, \quad \lambda_r = \sqrt{\left(\sqrt{1+\eta_r^2}-1\right)\Big/2} \tag{11}$$

The theorem of residues can now be applied to the contours C_1 and C_2 in Fig. 1 provided that $\delta \to 0$, $R \to \infty$. The contours avoid passing through the point at $\omega = 0$, where the integrand is not analytic.

The integral in Eq. (9) has been calculated in Gaul et al. (1985), given by:

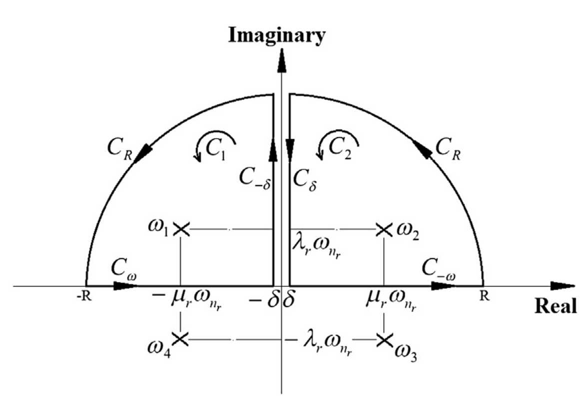

Fig. 1. Integral contours and location of poles.