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Analyzing contact problem between a functionally graded plate of finite dimensions and a rigid spherical indenter

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ABSTRACT

Elastic contact of a functionally graded plate of finite dimensions with continuous variation of material properties and a rigid spherical indenter is studied. The plate is consisted of a ductile (metal) phase at the lower and a brittle (ceramic) phase at the upper surface. The punch acts on the upper surface which is the ceramic richer section of the plate. The contact problem in functionally graded (FG) structures has been studied widely; in such problems the main focus has been on FG structures with infinite dimensions where Hertzian or modified Hertzian contact laws can properly predict the contact parameters such as the size of the contact region and the pressure distribution under the punch. However, due to the finite dimensions of the considered plate in this study, the contact problem needs to be reconsidered. While Hertz's contact law predicts a power equal to 1.5 for the force indentation relation, the results of this study show that for an FG plate the exponent of the contact law depends on the brittle to ductile phases ratio of moduli of elasticity and material properties distribution. In cases in which the brittle phase has a lower modulus of elasticity compared to the ductile phase, the contact law exponent is independent of material properties distribution. In addition, in such cases the maximum compressive contact stress is located directly on the upper surface of the plate. On the other hand, in cases in which the brittle phase is stiffer than the metal phase, the exponent of the contact law is a function of material properties distribution and the location of the maximum compressive contact stress is beneath the upper surface. In addition, in general the contact parameters are independent from the microstructural interactions of the constituting phases. Since several numerical examples are examined here, these findings can be interpreted as the most general rules in the contact problem between an FG plate and a rigid sphere.

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1. Introduction

Functionally graded materials (FGMs) are composite materials mostly consisted of metal and ceramic phases (Yu et al., 2010). These materials are generally designed for a special function which is mainly defined by considering the environment that the material is supposed to work in. In addition, the non-homogeneity concept which is accompanied by this group of materials enables the industrial engineers to design structures with optimum performance. A precious review on the concept, application and analysis of FGMs is presented by Birman and Byrd (2007). One of the most important applications of FGMs is in developing surfaces that can sustain contact loading conditions (Cooley, 2005). In addition, applying graded materials seems to be a solution in reducing the induced damage due to impact loading on structures (Apetre et al., 2006; Tasdemirci and Hall, 2009). On the other hand, in designing structures which are under impact loading the main initial step is to study the contact problem between the bodies (Apetre et al., 2006). Thus the study of contact is an important stage in designing a graded structure. The first analytical solution of the contact mechanics between two bodies has been presented by Hertz in 1881 (Hertz, 1881). After Hertz, the problem of contact mechanics has been studied widely by many researchers; an excellent range of examples and solutions is provided by Johnson (1985). However studying the contact between bodies is a difficult task since the contact problem is a mixed boundary value problem with known traction outside and displacement inside the contact region. Due to the non-homogeneity accompanied with the concept of FGMs, the contact problem in these materials is more complex. The contact problem of non-homogeneous and anisotropic beams and plates is briefly

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reviewed by Abrate (Abrate, 1998). Wu and Yen (1994) have worked on the contact of laminated composite plates and rigid spherical indenters (Wu and Yen, 1994). Anderson (2003) and Saadati and Sadighi (2011) have studied the contact problem in composite sandwich plates (Anderson, 2003; Saadati and Sadighi, 2011).

In the field of FGMs, the focus has been on the contact mechanics of graded coating/substrate systems. Giannakopoulos (1998) has addressed the analytical and experimental advances in indentation of FGMs in the absence of residual stresses (Giannakopoulos, 1998). Guler and Erdogan (2007) considered the contact problems of parabolic and cylindrical stamps on graded coatings with exponential variation of elastic properties (Guler and Erdogan, 2007). Choi and Paulino (2008) and Dag et al. (2012) have studied the same contact problem but for a flat punch (Choi and Paulino, 2008; Dag et al., 2012). Liu et al. (2012) have considered the heat generation due to sliding contact and have solved the coupled thermo-elastic contact problem (Liu et al., 2012).

An important structure which is frequently used by many engineers and designers is rectangular plates with finite dimensions. On the other hand, compared to the case of conventional fiber composite laminates, considering the benefits of FGMs has turned the idea of taking advantage of functionally graded plates as an attractive option for engineers and designers. However as mentioned above, the investigations in the field of the contact mechanics in FGMs are concentrated on the semi-infinite regions. But considering the influence of size effects on the contact parameters, particularly the effect of finite thickness, necessitates an independent study of the contact mechanics in a functionally graded plate of finite dimensions and thickness. The authors of this paper (Nikbakht et al. in 2012 and 2013) have developed an elastic analytical-numerical approach to explore the contact parameters of a functionally graded plate and a rigid spherical indenter which is validated by ABAQUS finite element package as well (Nikbakht et al., 2013a, 2013b). They have presented numerical examples for graded low carbon steel plates which are either coated on one surface or both surfaces by vitreous enamel. Their results show that for such graded plates the contact force is proportional to the indentation to the power of about 2.0 regardless of the variation in the material properties. In addition, they have numerically and experimentally examined the elastic–plastic indentation of a vitreous enameled low carbon steel plate by a rigid spherical indenter and found out that the best curve fitted on the force-indentation data is a polynomial of power 3.0 (Nikbakht et al., 2013c).

While the elastic–plastic contact must be studied for each material distribution and a general rule may not be extracted, the elastic contact problem seems to follow more regular laws. Besides, in many industrial applications such as those dealing with low velocity impact, considering elastic contact deformation is sufficient for engineering purposes. The previous studies conducted by the authors include a graded plate with a brittle phase (enamel) which is less stiff than the ductile phase (low carbon steel). Thus it is important to establish the contact force-indentation relation for a graded plate in a more general way.

In this research, the frictionless elastic contact mechanics of a functionally graded plate of finite dimensions and a rigid spherical punch is studied. The main idea is to extract a general decree that describes the effect of material properties variation on the contact problem of an FG plate and a rigid sphere. The plate is assumed to be simply supported and is richer from the brittle phase at the upper and from the ductile phase at the lower surface and the punch is in contact with the brittle phase. The variation of elastic material properties is considered to be one dimensional through the thickness of the plate and is estimated by a volume fraction based model originally proposed by Tamura, Tomota and Ozawa. The analysis method of the contact problem is based on the work of Nikbakht et al. (2013a). By using this analytical method, the effect of material properties variation on the contact law is studied for different ceramic to metal elastic modulus ratios. In addition, an industrial example is presented for aluminum-alumina graded plates and the results are compared to the previously obtained results by the authors for graded enameled low carbon steel plates.

2. Material properties

In most graded structures, the main available parameter of the microstructure is the distribution of the volume fraction of the constituting phases. Thus, the variation of the volume fraction should be the key parameter in any accurate model of estimating the effective material properties of a graded structure. On the other hand, the microstructural behavior and interactions of the constituting phases must be taken into account as well. In order to achieve these goals, in this research, a volume fraction based model originally proposed by Tamura, Tomota and Ozawa (called TTO model henceforth) (Tamura et al., 1973; Jin et al., 2003) is used to estimate the effective material properties of the graded plate. This model relates the uniaxial stress, σ, and strain, ε, of a two-phase composite to the corresponding average stresses and strains of the two constituent materials. For a two-phase composite:

$$\sigma = V_1\sigma_1 + V_2\sigma_2 \text{ and } \varepsilon = V_1\varepsilon_1 + V_2\varepsilon_2 \tag{1}$$

where σ_i and ε_i (i = 1,2) denote the average stresses and strains in the constituting phases. The TTO model introduces an additional parameter q, called the stress to strain transfer ratio which depends on the effect of microstructural behavior of the constituting phases and is defined as

$$q = \left|\frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2}\right|, \quad 0 \le q \le \infty \tag{2}$$

This parameter may be determined experimentally or numerically for any composite material system consisted of two phases. By assuming linear stress–strain relation of the constituting phases and by setting the index regarding the properties of the metal phase equal to 2, in elastic applications the TTO model estimates the modulus of elasticity of the composite structure, E, to be

$$E = \frac{V_2E_2(q+E_1)/(q+E_2) + (1-V_2)E_1}{V_2(q+E_1)/(q+E_2) + (1-V_2)} \tag{3}$$

where E_i (i = 1,2) stands for the modulus of elasticity of the constituting phases. As described before, the graded plate is coated on one side; therefore, a proper approximation of the volume fraction distribution through the thickness of the plate may be a power-law relation as

$$V_2 = \left(\frac{1}{2} - \frac{z}{h}\right)^r, \quad -\frac{h}{2} \le z \le \frac{h}{2} \tag{4}$$

where z is the coordinate in the gradation direction, h is the thickness of the graded medium (in the direction of gradation) and r is the exponent which describes the way in which the volume fraction distributes in the graded layer. By using nano-indentation technique, Nikbakht et al. has examined and verified the appropriate efficiency of this model in predicting effective elastic material properties of FGMs compared to ordinary models such as linear rule of mixtures (Nikbakht et al., 2013b).

In this research a range of ceramic to metal elastic modulus ratios (E_1/E_2) is considered to find the effect of changing this ratio on the contact parameters. In addition, the results for analyzing the contact problem for two industrial examples are presented and

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