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Dynamic stability of a rotating sandwich beam with magnetorheological elastomer core

B. Nayak [a,b], S.K. Dwivedy [b,*], K.S.R.K. Murthy [b]

[a] Mechanical Engineering Department, Centurion University of Technology and Management, Odisha 752050, India
[b] Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, India

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ABSTRACT

In this work the dynamic stability of a rotating three layered symmetric sandwich beam with magnetorheological elastomer (MRE) core and conductive skins subjected to axial periodic loads has been investigated using finite element method (FEM). The derived governing equation of motion is in the form of a multi-degree of freedom Mathieu–Hill's equation with complex coefficients. The instability regions of the sandwich beam for the principal parametric resonance case have been determined by using the harmonic balance method. Effects of applied magnetic field, rotating speed, setting angle, hub radius, static load and dynamic load on the dynamic characteristics and instability regions of the sandwich beam are investigated. This work will find application in the passive and active vibration reduction of rotating sandwich structure using magnetorheological elastomer core, magnetic field and periodic axial load.

1. Introduction

Various smart materials are used in structures for a wide range of applications and also for active and passive vibration control. Materials with controllable rheological properties, such as magnetorheological elastomers (MRE), electrorheological (ER) and magnetorheological fluids, are promising for many applications including adaptive tuned vibration absorbers (ATVA), variable stiffness suspension bushing, variable impedance surfaces, prosthetic devices, tunable automotive mounts and structures (Watson and Canton, 1997; Ginder et al., 1999; Carlson and Jolly, 2000; Deng et al., 2006; Guðmundsson, 2011). Magnetorheological elastomers (MRE) have great potential in developing stiffness variable devices which can find applications in many intelligent structures. The mechanical properties of MREs such as shear modulus and damping factor can be changed and controlled by varying magnetic field (Kallio, 2005; Zhang et al., 2010; Boczkowska et al., 2012).

Many works have been reported on development of new MREs based on natural and synthetic rubbers by many researchers for improving the MR effect and mechanical properties (Davis, 1999; Sun et al., 2008; Zajac et al., 2010; Kaleta et al., 2011). Chen et al. (2007) developed natural rubber based MREs by considering different percentage of iron particles to improve the mechanical properties and reported that the increase in weight fraction of iron particles in natural rubber increases the shear modulus of MRE. Later Chen et al. (2008) added nano particles of carbon black in the natural rubber in addition to iron particles to fabricated MREs with high shear modulus, low loss factor and improved tensile strength.

In recent years MREs have been used to carry out vibration analysis to observe the controllable properties and the vibration suppression capabilities of three-layered MR adaptive sandwich beams with suitable magnetic field (Sun et al., 2003; Li and Gong, 2008; Lara-Prieto et al., 2010; Hu et al., 2011). It has been observed that the vibration suppression capabilities of MRE adaptive structures are significant. In sandwich beams, the core goes through considerable shear strain as the structure deforms. The core can dissipate the energy and consequently suppress the vibration response. Many works have been reported on free and forced vibration of sandwich beams with viscoelastic cores using various approaches and different methods (Mead and Markus, 1969; Rao, 1978; Howson and Zare, 2005; Banerjee et al., 2007; Qin and Wang, 2009). Zhou and Wang (2005, 2006a, 2006b) studied the dynamic properties of sandwich beams with MRE embedded soft cores with nonconductive and conductive skins using higher order theory. Recently Nayak et al. (2012b) investigated the effect of location and length of MRE patch in the core on dynamic characteristics of an MRE embedded viscoelastic cored sandwich beam with conductive skins.

Parametric instability can occur when any of the terms of the homogenous part of the equations of motion of a system have periodically varying coefficients, i.e. the loading, stiffness, inertia,

* Corresponding author. Tel.: +91 361 2582670; fax: +91 361 2582699.
E-mail address: dwivedy@iitg.ernet.in (S.K. Dwivedy).

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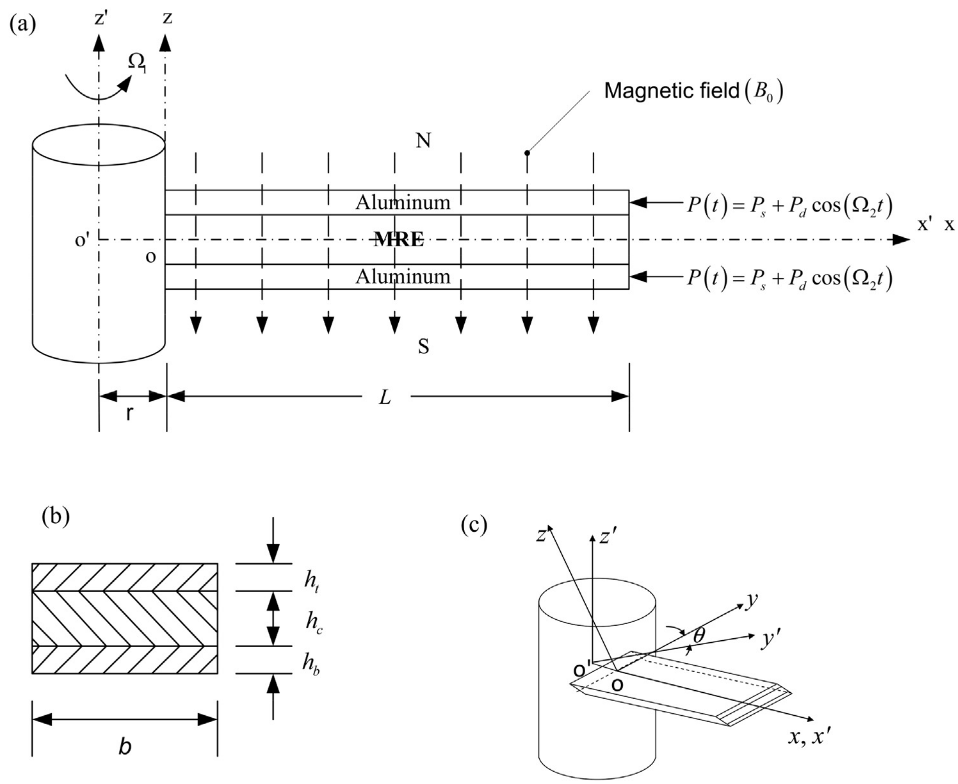

Fig. 1. (a) Rotating sandwich beam with MRE core subjected to magnetic field and periodic axial load (b) cross section of the sandwich beam (c) setting angle definition.

and damping-terms. A simple example of a parametrically excited system is a beam subjected to time varying axial load (Nayfeh and Mook, 1979) where the beam starts to buckle and vibrate for certain amplitude and frequency of the axial load even though the axial load is well below the critical Euler buckling load. In these systems one should study the parametric instability regions to obtain the critical system parameters to avoid excessive vibration of the system. The dynamic instability of beams subjected to a time varying axial load was presented by Bolotin (1964). The dynamic instability of sandwich structures induced by periodic axial force has been investigated by many researchers (Kar and Sujeta, 1991; Ray and Kar, 1995, 1996). Nayak et al. (2011, 2012a). investigated the instability regions of an MRE embedded viscoelastic cored sandwich beam subjected to periodic axial load for various resonance cases.

The study of dynamic characteristics of rotating beams is very important in design of -rotating structural elements. Such beams may be subjected to axial load (Kar, 1986; Sakar and Sabuncu, 2003; Sabuncu and Evran, 2005; Sinha, 2005) and for reduction of vibration and to make it lightweight one may use sandwich beams. Sandwich beams are in use for many years in structural engineering such as aeroplanes, helicopter rotor blades, and robot arms as a load carrying member with high strength to weight ratio. Many works have been reported on free and forced vibration of rotating beams using various approaches and different methods (Putter and Manor, 1978; Bhat, 1986; Lee, 1994). Hoa (1979) investigated the free vibration of a rotating beam with tip mass considering different system parameters. Abbas (1986) determined the instability regions of a rotating Timoshenko beam by finite element method. Bulut (2013) studied the effect of taper ratio on the stability of a rotating tapered beam with periodically varying speed. Wei et al. (2006) studied the vibration control of a flexible rotating sandwich beam with electrorheological (ER) core. Lin and Chen (2003) studied the dynamic behavior and dynamic instability of a rotating beam with a constrained damping layer by finite element method. It may be noted that some studies have been carried out on rotating sandwich beams with viscoelastic core. The vibration analysis of a rotating sandwich beam with MRE core and conductive skins however, has not yet been explored.

In this present study, the governing equation of motion a rotating sandwich beam with MRE core and conductive skins subjected to time varying axial load is developed using finite element method. The validity of modal frequencies of rotating sandwich beam is demonstrated by comparing with published results. The effects of applied magnetic field, rotating speed, setting angle, hub radius, static load and dynamic load on the dynamic characteristics and instability regions of the sandwich beam are investigated.

2. Dynamic model of an MRE based rotating sandwich beam

Fig. 1 shows a rotating sandwich beam where the Cartesian co-ordinate system x, y, z has the origin at O i.e., at the root of the sandwich beam with orientation along the major dimensions of the beam. Also, the Cartesian co-ordinate system x', y', z' has the origin at the hub center O' about which the beam rotates. The rotating sandwich beam with MRE core and conductive aluminum skins is subjected to magnetic field (B_0) and time varying axial force ($P(t) = P_s + P_d\cos(\Omega_2 t)$) as shown in Fig. 1(a). Here P_s and P_d are the static and dynamic load respectively, t is the time and Ω_2 is the excitation frequency. The beam has a length L and width b. The top, bottom and core layers thickness are h_t, h_b and h_c, respectively as shown in Fig. 1(b). The setting angle θ is the angle between the mid-plane of the sandwich beam and the plane of rotation as shown in Fig. 1(c) which is similar to that shown in the work of Hoa (1979). r is the hub radius. In this formulation the longitudinal displacements of the mid-planes of the top, bottom and core layers in the x-direction are u_t, u_b and u_c respectively.