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A continuum-based mixed shell element for shakedown analysis

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The objective of this paper is to present a new triangular shell element for shakedown analysis. The element is based on a mixed variational formulation with displacement/velocity and stress fields independently interpolated. We also employ the continuum based (CB) approach which allows the definition of the yield function at the continuum level without any approximation in terms of generalized stresses. The formulation of the proposed element is presented in details after a brief review of relevant classical shakedown theory concepts. The performance of the element is assessed by means of a set of selected representative examples. The numerical tests include: (i) analyses of thin and thick-walled straight pipes under combined loads, (ii) the shakedown analysis of a pipe bend under internal pressure and in-plane bending and (iii) limit analysis of a cylinder–cylinder intersection subjected to bending and internal pressure.

1. Introduction

The safety of structures subjected to variable loading can commonly be assessed by means of shakedown analysis. In the classical shakedown theory (Debordes and Nayroles, 1976; Kamenjarzh, 1996; Koiter, 1960; Nguyen, 2000) the structure material is assumed linear elastic-perfectly plastic and the assessment procedure involves finite element discretizations of the structure and the use of dedicated numerical procedures for the solution of the shakedown analysis problem (Maier et al., 2003).

In the case of thin structures, three-dimensional solid finite element models can be computationally very expensive due to the large number of elements required. Coarser meshes, on the other hand, are prone to contain elements with high aspect ratios which in turn can cause ill conditioning of the finite element equation system and lead to loss of accuracy in the solution. These problems are generally circumvented by using structural elements such as beam, plate and shell elements. However, for limit and shakedown analysis, the use of classical structural elements requires the development of a solution approach with laborious approximations in terms of generalized stresses to express, for instance, the yield limit of the material (Maier et al., 2003).

The contribution of this paper is the development of a shell element for shakedown analysis of 3D structures which, in particular, bypasses the need to express the yield limit in terms of generalized stresses. The proposed element is formulated using the continuum based (CB) approach (Ahmad et al., 1970; Belytschko et al., 2001; Benson et al., 2010; Buechter and Ramm, 1992; Hughes and Liu, 1981a, 1981b; Hughes, 1987; Simo and Fox, 1989) and a mixed Hellinger-Reissner-type formulation (Bathe, 1996; Han and Reddy, 1999; Christiansen, 1996) with stress and displacement/velocity fields interpolated independently.

In our opinion, the use of mixed elements in this context is advantageous because they can be more easily implemented. In addition, for kinematically-based shell formulations (Belytschko et al., 2001) the use of the kinematic minimum principle for shakedown analysis requires the computation of the dissipation function, which is unbounded unless plastic strain rates are plastically admissible at all points of the element. This introduces a very stringent constraint in non-constant plastic strain interpolations.

In the present development we focus our attention on the ability of the finite element to efficiently represent solutions of the considered problem, with special emphasis on the accurate determination of critical amplification factors for the prescribed reference domain of variable loadings. The numerical procedure used

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here for the general shakedown problem was presented in Zouain et al. (2002), Zouain (2004).

A mixed discretization of the continuum formulation of shakedown analysis leads to an approximate problem with augmented number of unknowns (velocity and stress parameters) in comparison to purely kinematical approaches. However, this discrete min–max problem is equivalent to two convex optimization problems and this fact is exploited in the numerical procedures commonly used in shakedown analysis (see e.g. Zouain (Zouain, 2004)) with the result that a unique algorithm is implemented to solve both types of problems with identical computational cost.

The aim of obtaining critical load approximations in limit analysis that could be surely classified as strict lower or upper bounds has been frequently pursued by many authors. This objective is attractive from a practical point of view, mostly because lower bounds lead to true conservative safety assessment, and also from a computational perspective, because errors on the load factor can be bounded rather than estimated. In the case of 2D or 3D continuum models there are successful discretizations attaining strict lower and upper bounds, due to several authors, some of which are cited in Zouain et al. (2014) with corresponding key references. Differently, mixed finite elements are meant to be efficient but they do not exhibit bounding properties, except for very particular interpolations, which become then purely kinematical or strictly equilibrated and plastically admissible (Christiansen, 1996). In the context of 2D continuum models, a discussion of bounding inequalities relating a family of mixed, kinematical and statical triangular elements was given by Zouain et al. (Zouain et al., 2014). In the case of shells, additional issues arise, such as the approximations concerning the shell geometry, that make strict bounds extremely difficult to achieve.

We remark that the proposed CB shell element differs from the family of mixed two-field triangular shell elements of Bathe and co-workers (Bathe and Lee, 2011; Kim and Bathe, 2009; Lee and Bathe, 2004) where displacement and strain fields are interpolated and combined in a particular way with the main objective of avoiding locking, usually present in pure displacement formulations. The element developed in this paper also differs from the mixed (strain-displacement) triangular shell element originally proposed by Argyris and co-workers (Argyris et al., 2000, 2003, 1998, 2002) and further modified by Corradi and Panzeri (Corradi and Panzeri, 2003, 2004) for application in sequential limit analysis.

The examples of applications presented in the article were selected aiming to allow comparison with published results pertaining to the field of shakedown analysis applied to shells. The comparison of the proposed CB shell element to other finite elements available in the literature when used to solve linear or nonlinear incremental analysis would be also worth to accomplish (on a much wider set of published results) but this is beyond the scope of the paper.

Another remark concerning the choose and evaluation of the examples of application is worth pointing out: In general, continuum based interpolations are not directly linked to any particular shell theory, except for the conditions used to relate master and slave degrees of freedom. Consequently, numerical results from CB shells should be directly compared to analytical or numerical solutions corresponding to a solid model of the shell. Nevertheless, analytical or numerical known solutions to the differential equations of a particular shell theory are also used in practice whenever the example closely fits the basic assumptions of the theory (or even when no better solution is available for comparison).

The paper is structured as follows: We first present in brief the concepts of the classical shakedown theory. Then we describe in detail the theory and implementation of the CB shell element. Next, the performance of the element in problems of limit and shakedown analysis is illustrated by means of representative numerical tests. The paper then closes with the presentation of concluding remarks.

2. Classical shakedown theory

In this section we provide a brief review of the classical shakedown theory (Koiter, 1960; Debordes and Nayroles, 1976), mainly following (Zouain, 2004). In particular we present here: (i) the formal definition of shakedown; (ii) the basic theorems of the classical theory; and (iii) the set of equations to be solved in shakedown problems both in the continuum and the discrete setting.

Structures made of a perfectly plastic material can develop three modes of failure when subjected to a combination of fixed and variable loads. Namely, alternating plasticity (plastic shakedown), incremental collapse (ratcheting), or instantaneous collapse (plastic collapse).

The structure undergoes alternate plasticity when the fluctuating loading program produces, after any arbitrarily large time, some plastic deformation, as well as a subsequent vanishing of the net plastic deformation. This induces failure due to low cycle fatigue. Likewise, the structure fails by incremental collapse when plastic deformations accumulate in the form of a compatible strain distribution that leads to excessive inelastic deformation. Instantaneous collapse takes place when a non-fluctuating load produces kinematically admissible plastic strain rates under constant stresses. This later phenomenon can be seen as a particular case of incremental collapse (for zero amplitude of load fluctuations) (Maier et al., 2003; Zouain, 2004).

Classical shakedown theory deals with the prevention of any of the aforementioned phenomena. Its main objective is the computation of the load amplification factor, μ, ensuring elastic adaptation (elastic shakedown). The computation of the amplification factor that only prevents instantaneous collapse is particularly called limit analysis.

Shakedown theory allows working under the realistic assumption that only the range of variable loading is known. The procedures developed under this theory, collectively named direct methods, make it possible the direct computation of the amplification factor without the need of a theoretically generally infinite number of full incremental analyses. Moreover, only the theory of direct methods can answer whether critical loads or cycles do exist, or not, independently from load histories.

2.1. Basic notation

2.1.1. Kinematics and equilibrium

Consider a body occupying an open bounded region $\mathscr{B}$ with a regular boundary Γ. The set of all admissible velocity fields $\hat{\boldsymbol{v}}$ complying with homogeneous boundary conditions prescribed on the part $\Gamma_{\boldsymbol{u}}$ of Γ is the space $\mathcal{V}$. Strain rate tensor fields $\hat{\boldsymbol{d}}$ are elements of $\mathcal{W}$, and the tangent deformation operator $\mathcal{D}$ maps $\mathcal{V}$ into $\mathcal{W}$. The dual space of $\mathcal{W}$ is the space $\mathcal{W}'$ of stress fields $\hat{\boldsymbol{\sigma}}$. The equilibrium operator $\mathcal{D}'$, dual of $\mathcal{D}$, maps elements of $\mathcal{W}'$ into the space of load systems $\hat{\boldsymbol{F}}$ denoted by $\mathcal{V}'$. Prescribed loads $\hat{\boldsymbol{F}}$ vary quasi-statically. The spaces $\mathcal{V}$ and $\mathcal{V}'$ are also dual. Consequently, static and kinematic relations are written as

$$\hat{\boldsymbol{d}} = \mathcal{D}\hat{\boldsymbol{v}}, \quad \hat{\boldsymbol{F}} = \mathcal{D}'\hat{\boldsymbol{\sigma}}. \tag{1}$$

Moreover, the hypothesis of small deformation holds and the infinitesimal displacement and strain fields, $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{\varepsilon}}$ respectively, are related by