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Exact solution for an interface crack between dissimilar anisotropic thermoelastic solids under uniform heat flow

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ABSTRACT

An interface crack between dissimilar anisotropic thermoelastic solids subjected to uniform heat flow at infinity and no mechanical loading on its boundary is analyzed. Generalized two-dimensional deformations such as plane stress and plane strain deformations are considered. The exact solution for the thermoelastic fields is obtained on the basis of the thermoelastic formalism with the transformed function representations. The closed-form solutions of the stress intensity factor and energy release rate are obtained, and the explicit exact expressions for the stress intensity factor and energy release rate are derived for both orthotropic and isotropic bimaterials. Numerical computations are performed using the finite element method to verify the exact solutions. Close agreement is observed between the exact solution and the numerical solution.

1. Introduction

Analytic solutions for cracks in an elastic solid have received much attention owing to their potential application in various kinds of fracture problems. They provide a better understanding of the distributions of elastic fields and shed light on the physical effects of material constants and geometric parameters. The use of the exact solutions also makes it possible to verify a new numerical technique.

Layered structures composed of dissimilar materials have found a variety of engineering applications in the fields of electronic devices, semiconductors and optical electronics. There have been numerous studies on a finite interface crack embedded in an infinite bimaterial as a basic problem. Rice and Sih (1965) solved an interface crack in dissimilar isotropic solids subjected to remotely uniform stresses and obtained the exact solution of the stress intensity factor. Suo (1990) studied the problem of interface cracks between dissimilar anisotropic media, and Beom and Atluri (1996) investigated interface cracks in an anisotropic piezoelectric bimaterial. Thermal stresses induced in bimaterials under thermal loading can cause growth of interface cracks, which often results in premature structure failures. Lee and Shul (1991) solved an insulated interface crack in an isotropic thermoelastic bimaterial by using the complex variable method (Bogdanoff, 1954). Hwu (1990) derived the extended Stroh formalism for anisotropic thermoelasticity on the basis of the Stroh formalism (Stroh, 1958). Subsequently, Hwu (1992) solved interface crack problems in dissimilar anisotropic thermoelastic media by applying the extended Stroh formalism. Banks-Sills and Dolev (2004) and Khandelwal and Chandra Kishen (2009) numerically investigated interface cracks in an isotropic thermoelastic bimaterial. Li and Kardomateas (2006) solved an interface crack in an anisotropic thermoelastic bimaterial on the basis of the extended Stroh formalism. However, the correct exact solution of the stress intensity factor for an interface crack in dissimilar isotropic or anisotropic thermoelastic solids is not yet resolved in the explicit form (Banks-Sills and Dolev, 2004). Hence, the problem of an interface crack in a thermoelastic bimaterial is revisited in this paper. The extended Stroh formalism is not valid for a degenerate anisotropic material with multiple thermoelastic characteristic roots. Recently, Beom (2013a,b) developed a new anisotropic thermoelastic formalism based on the transformed function representation, which does not break down for a degenerate anisotropic solid.

The purpose of this study is to investigate an interface crack between dissimilar anisotropic thermoelastic solids with infinite extents. The anisotropic bimaterial is subject to uniform heat flow at infinity under steady-state conditions, and the crack surface is assumed to be thermally insulated. No mechanical loading is applied to its boundary. The interface crack problem is formulated on the basis of the transformed function representations for plane solutions of anisotropic thermoelastic solid, (Beom, 2013a) and the exact solution for thermoelastic fields is obtained in the integral form. Special attention is paid to the interface stress intensity factor and energy release rate for the interface crack. The interface stress intensity factor and energy release rate for an anisotropic

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bimaterial are derived in the closed form. In particular, the exact expressions of the interface stress intensity factor and energy release rate for orthotropic and isotropic bimaterials are explicitly obtained, and the effects of thermal load and material parameters are discussed. To verify the exact solution for the stress intensity factor and energy release rate, numerical computations are performed by using the finite element method.

2. Formulation

Consider an anisotropic solid subjected to an arbitrary two-dimensional thermal condition on its surface. The temperature field depends only on the in-plane coordinates x_1 and x_2. Under steady-state conditions, two-dimensional thermal fields satisfying the heat conduction equation can be written as (Nowacki, 1962)

$$\begin{aligned} &T = 2\mathrm{Re}[\Omega'(z_0)], \\ &\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = -2\mathrm{Re}\left[ik\Omega''(z_0)\begin{Bmatrix} -p_0 \\ 1 \end{Bmatrix}\right]. \end{aligned} \tag{1}$$

Here, T is the temperature change from a reference temperature for an unstressed state, q_i is the heat flux, $\Omega(z_0)$ is the thermal potential function, and $z_0 = x_1 + p_0 x_2$. p_0 and k are given by

$$\begin{aligned} &p_0 = \lambda_0^{-\frac{1}{2}}\left(i\sqrt{1-\nu_0^2} - \nu_0\right), \\ &k = \sqrt{k_{11}k_{22} - k_{12}^2}, \end{aligned} \tag{2}$$

where k_{ij} is the symmetric coefficient of heat conduction,

$$\begin{aligned} &\lambda_0 = \frac{k_{22}}{k_{11}}, \\ &\nu_0 = \frac{k_{12}}{\sqrt{k_{11}k_{22}}}. \end{aligned} \tag{3}$$

Recently, Beom (2013a) derived transformed function representations for plane solutions of an anisotropic thermoelastic solid. The thermoelastic formalism is valid for a degenerate anisotropic solid with multiple thermoelastic characteristic roots as well as for an anisotropic solid with distinct thermoelastic characteristic roots. According to the thermoelastic formalism (Beom, 2013a), a general solution of elastic fields that satisfies the equilibrium equation for an anisotropic thermoelastic solid under generalized two-dimensional deformation can be written as follows:

$$\begin{aligned} &\{u_i\} = 2\mathrm{Re}\left[-i\mathbf{H}\left(\boldsymbol{\phi}(x_1,x_2) + \boldsymbol{\phi}^0(x_1,x_2)\right) + \Omega(z_0)\mathbf{c}^0\right], \\ &\{\sigma_{1i}\} = -2\mathrm{Re}\left[\frac{\partial}{\partial x_2}\left(\boldsymbol{\phi}(x_1,x_2) + \boldsymbol{\phi}^0(x_1,x_2)\right)\right], \\ &\{\sigma_{2i}\} = 2\mathrm{Re}\left[\frac{\partial}{\partial x_1}\left(\boldsymbol{\phi}(x_1,x_2) + \boldsymbol{\phi}^0(x_1,x_2)\right)\right] \quad (i = 1,2,3). \end{aligned} \tag{4}$$

Here, u_i is the displacement, σ_{ji} is the stress, and Re denotes the real part. $\boldsymbol{\phi}$, $\boldsymbol{\phi}^0$, and $\mathbf{c}^0$ are given by

$$\begin{aligned} &\phi_i = \sum_{j,k=1}^{3} B_{ij}B_{jk}^{-1}g_k(z_j), \\ &\phi_i^0 = -\sum_{j,k=1}^{3} B_{ij}B_{jk}^{-1}d_k\Omega(z_j) + d_i\Omega(z_0), \\ &\mathbf{c}^0 = i\mathbf{Hd} + \mathbf{c}, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\mathbf{B} = \begin{bmatrix} -p_1 & -p_2 & -p_3\xi(p_3) \\ 1 & 1 & \xi(p_3) \\ -\eta(p_1) & -\eta(p_2) & -1 \end{bmatrix}, \\ &\mathbf{d} = \frac{\chi(p_0)}{N(p_0)}\begin{Bmatrix} -p_0 \\ 1 \\ -\zeta(p_0) \end{Bmatrix}, \\ &\mathbf{c} = \frac{\chi(p_0)}{N(p_0)}\begin{Bmatrix} A_1^0(p_0) \\ A_2^0(p_0) \\ A_3^0(p_0) \end{Bmatrix} + \frac{1}{p_0}\begin{Bmatrix} p_0\alpha_1^0 \\ \alpha_2^0 \\ \alpha_4^0 \end{Bmatrix}. \end{aligned} \tag{6}$$

Here, $g_i(z)$ $(i = 1, 2, 3)$ are complex functions in which $z = x_1 + px_2$, where p is a complex number with a positive imaginary part, $z_i = x_1 + p_i x_2$, and p_i $(i = 1, 2, 3)$ are the roots with a positive imaginary part to the characteristic equation $N(p_i) = 0$. The functions $N(p)$, $\xi(p)$, $\eta(p)$, $\chi(p)$, and $A_i^0(p)$ $(i = 1, 2, 3)$ are

$$\begin{aligned} &N(p) = \ell_2(p)\ell_4(p) - [\ell_3(p)]^2, \\ &\xi(p) = -\frac{\ell_3(p)}{\ell_4(p)}, \\ &\eta(p) = -\frac{\ell_3(p)}{\ell_2(p)}, \\ &\chi(p) = \ell_2^0\ell_2(p) - \ell_1^0\ell_3(p), \\ &A_1^0(p) = S_{11}p^2 + S_{12} - S_{16}p + \zeta(p)(S_{15}p - S_{14}), \\ &A_2^0(p) = S_{21}p + \frac{S_{22}}{p} - S_{26} + \zeta(p)\left[S_{25} - \frac{S_{24}}{p}\right], \\ &A_3^0(p) = S_{41}p + \frac{S_{42}}{p} - S_{46} + \zeta(p)\left[S_{45} - \frac{S_{44}}{p}\right], \end{aligned} \tag{7}$$

$$\begin{aligned} &\ell_2(p) = S_{55}p^2 - 2S_{45}p + S_{44}, \\ &\ell_3(p) = S_{15}p^3 - (S_{14} + S_{56})p^2 + (S_{25} + S_{46})p - S_{24}, \\ &\ell_4(p) = S_{11}p^4 - 2S_{16}p^3 + (2S_{12} + S_{66})p^2 - 2S_{26}p + S_{22}, \\ &\ell_1^0 = -\alpha_5^0 p_0 + \alpha_4^0, \\ &\ell_2^0 = -\alpha_1^0 p_0^2 + \alpha_6^0 p_0 - \alpha_2^0, \\ &\zeta(p) = \frac{\ell_1^0 N(p) - \chi(p)\ell_3(p)}{\chi(p)\ell_2(p)}. \end{aligned} \tag{8}$$

Here, $\alpha_1^0 = \alpha_{11}^0, \alpha_2^0 = \alpha_{22}^0, \alpha_4^0 = 2\alpha_{23}^0, \alpha_5^0 = 2\alpha_{13}^0$, and $\alpha_6^0 = 2\alpha_{12}^0$, in which α_{ij}^0 is the coefficient of thermal expansion. Eqs. (4)–(8) are valid for plane stress deformation ($\sigma_{33} = 0$). For plane strain deformation ($\varepsilon_{33} = 0$), S_{ij} and α_i are replaced with S_{ij}^* and α_i^*, respectively, where

$$\begin{aligned} &S_{ij}^* = S_{ij} - \frac{S_{i3}S_{j3}}{S_{33}}, \\ &\alpha_i^* = \alpha_i^0 - \frac{S_{i3}}{S_{33}}\alpha_3^0, \end{aligned} \tag{9}$$

where $\alpha_3^0 = \alpha_{33}^0$. $\mathbf{H}$ is the Hermitian matrix given by

$$\mathbf{H} = \mathbf{L} + i\mathbf{M}, \tag{10}$$

where $\mathbf{L}$ and $\mathbf{M}$ are symmetric and antisymmetric real matrices, respectively. The explicit expression of $\mathbf{H}$ can be found in Wei and Ting (1994). Once the heat conduction problem is solved for $\Omega(z)$