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An assessment of the role of the third stress invariant in the Gurson approach for ductile fracture

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ABSTRACT

The present paper is concerned with the effects of the Lode angle (or the third stress invariant) in the yielding of porous materials. This is addressed in the framework of Gurson's analysis of voided materials. It is shown first that without the approximations operated by Gurson, the Lode angle of the macroscopic strain rate is naturally involved in the analysis and consequently the third stress invariant affects the yield criterion. Pushing forward Gurson's analysis without his approximations and still considering his axisymmetric trial velocity field, changes on the predicted yield locus are observed but only in the intermediate range of triaxialities. The same predictions are obtained for very small and very large stress triaxialities. A more carefull inspection of the results shows in fact that most of the changes are second order effects and direct effects of the Lode angle are hardly visible.

Pursuing the analysis with a new class of trial velocity fields due to Si et al. (2007) (containing the Gurson field) and including non-axisymmetric contributions, changes on the prediction of the yielding behaviour are observed at the low triaxiality regime. Again, effects of the Lode angle are seen to be rather small at least in the range of porosities pertinent to ductile fracture. However, inclusion of Lode angle brings changes on the shape but also on the symmetries of the yield domain. The exact hydrostatic prediction obtained in the Gurson model is also maintained here. Comparison with the numerical simulations presented by Thoré et al. (2011) using optimization algorithms in the context of limit analysis show a very good agreement with the results and in particular that these results fall well between the static and kinematic limit analysis bounds that they furnished.

1. Introduction

The modelling of the behaviour and failure of ductile porous media has been and is nowadays again the subject of intense researches in non linear mechanics of materials. In their pioneering analyses, McClintock (1968) and Rice and Tracey (1969) studied the growth of long cylindrical voids and spherical voids respectively and showed that the void growth in ductile metals is strongly dependent on the hydrostatic stress. A decade later, Gurson (1977) proposed a yield criterion for voided materials using a limit analysis approach combined to a homogenization procedure and clearly explicited the role of the hydrostatic stress on the yielding behaviour of porous materials. Their observations have made the background for several fracture models still used today. However, there are a number of experimental situations where the above theories are shown to be unable to reproduce the observations. The most important ones are those where susceptibility to shear failure under low or negative triaxialities (McClintock, 1971; Teirlinck et al., 1988; Johnson and Cook, 1985; Bao and Wierzbicki, 2004; Barsoum and Fakeslog, 2007; Bai and Wierzbicki, 2008; Nahshon and Hutchinson, 2008; Fourmeau et al., 2013) is observed. Shear-dominated stress states such as plugging failure in projectile penetration are other examples (Børvik and et al., 2001) and many others can be found in the above references.

A number of improvements to the Gurson model exist in the literature. The Tvergaard refinement of the model (Tvergaard, 1981, 1982, 1990) including possible void interactions and the Gurson–Tvergaard–Needleman (Tvergaard and Needleman, 1984) model are probably the most well known. But several other contributions, addressing different issues, attempted to improve the Gurson model: an account of void shape effects is given in Gologanu et al. (1997), Gologanu et al. (1993), Gologanu et al. (1994), Garajeu et al. (2000) and Monchiet et al. (2006), the role of plastic anisotropy is studied in Benzerga and Besson (2001), the extension to pressure sensitive matrix behaviour is analysed in Thoré et al. (2009). The

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above contributions did not consider possible effects of the Lode angle.

It is interesting to note that the mechanics communities of soil, rock and concrete materials also used for some time conventional plasticity models for pressure sensitive materials (see e.g. Roscoe and Burland, 1968) formulated only in terms of the first and second stress invariants. But following the evidence of experimental observations on these materials (Chen and Saleeb, 1982; Lade and Duncan, 1975; Matsuoka and Nakai, 1974; Willam and Warnke, 1975), they recognized a long time ago how the third invariant of stress can affect significantly the fracture of these materials. This had led to a number of phenomenological macroscopic plasticity models involving the three stress invariants, see the review by Bardet (1990). These materials can, in general, withstand higher deviatoric stresses when subjected to confinement. There is yet another field where the third invariant of stress was observed to be an important parameter, this is the field of shape memory alloys where the transformation surfaces are seen to be affected by the Lode angle (Lavernhe Taillard et al., 2009).

The objective of this paper is twofold: on one side to introduce consistently the third invariant of the stress (or the Lode angle) in yielding and fracture of ductile materials and this is first carried out here in the framework of the Gurson analysis using his axisymmetric trial velocity field. And second, to possibly improve the predictions of the Gurson model, still including the Lode angle, by using another class of trial velocity fields proposed by Si et al. (2007) in a similar way as Monchiet et al. (2006) did, using the exterior field (to the inclusion) provided by Eshelby (1957) for the infinite space.

The Lode angle effects have been included and studied in a consistent way by Danas et al. (2008), see also Danas and Ponte Castañeda (2009) in an alternative approach to limit analysis of unit cell and based on second order variational homogenization techniques, which provide non linear Hashin–Shtrikman type upper bounds of the exact yield surfaces. The representative elementary volume, considered in such approach, consists of a rigid-plastic solid matrix containing ellipsoidal cavities. The results of Danas et al. (2008) will be compared to our results in this paper.

In Section 2, we present the general framework for the theory of plasticity of voided materials involving the three stress invariants in the case where the constitutive behaviour of the matrix is considered isotropic. We then concisely review the Gurson model and its derivation mainly in order to recall and emphasize the assumptions upon which it is built. Still using his trial velocity field, we present and discuss a parametric representation of the yield criterion involving the three stress invariants. In Section 4 an extended version of the Gurson model including the third invariant of stress is developed using a more general velocity trial field provided by Si et al. (2007). The last section thoroughly discusses the results and compare them to the numerical simulations by Thore et al. in the framework of limit analysis.

2. Isotropic plasticity theory of voided materials

2.1. Preliminaries

For voided materials with isotropic matrix behaviour, the general theory of plasticity indicates a macroscopic yield function dependent on the three stress invariants of the macroscopic stress tensor $\mathbf{\Sigma}$. The first and second invariants define respectively the hydrostatic stress $\Sigma_m = \Sigma_{kk}/3$, and the effective von Mises stress, $\Sigma_{eq} = \sqrt{3}J_2 = \sqrt{3/2S_{ij}S_{ij}}$, where $S_{ij} = \Sigma_{ij} - 1/3\Sigma_{kk}\delta_{ij}$ are the components of the stress deviator $\mathbf{\Sigma}'$ while the third invariant of stress is usually defined by

$$J_3 = \det\mathbf{\Sigma}' = S_{ij}S_{ik}S_{jk} = (\Sigma_1 - \Sigma_m)(\Sigma_2 - \Sigma_m)(\Sigma_3 - \Sigma_m) \tag{1}$$

where the expression on the right is in terms of the principal stresses, assumed to be ordered as $\Sigma_1 \geq \Sigma_2 \geq \Sigma_3$. Beside J_3, various equivalent measures are classically used to describe effects of the third stress invariant. One of these is the Lode angle defined by

$$\Theta = \frac{1}{3}\arccos\left(\frac{27}{2}\frac{\det\mathbf{\Sigma}'}{\Sigma_{eq}^3}\right) \tag{2}$$

and lying in the range $0 \leq \Theta \leq \pi/3$.

The Lode angle Θ allows to write the stress deviator (in its principal frame) as

$$\mathbf{\Sigma}' = \frac{2}{3}\Sigma_{eq}\begin{pmatrix} \cos\Theta & 0 & 0 \\ 0 & \cos(\Theta - \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\Theta + \frac{2\pi}{3}) \end{pmatrix} \tag{3}$$

and therefore the principal deviatoric stresses as

$$\begin{aligned} &\Sigma_1' = \frac{2}{3}\Sigma_{eq}\cos\Theta, \quad \Sigma_2' = \frac{2}{3}\Sigma_{eq}\cos\left(\Theta - \frac{2\pi}{3}\right), \\ &\Sigma_3' = \frac{2}{3}\Sigma_{eq}\cos\left(\Theta + \frac{2\pi}{3}\right) \end{aligned} \tag{4}$$

2.2. Stress invariants and some general consequences

Invoking isotropy, the macroscopic dissipation and any of its upper bounds that will be considered in the sequel are assumed to depend on the three strain rate invariants defined in a similar manner as those of the stress in Section 2.1. These invariants are usually defined by

$$\dot{E}_m = \frac{1}{3}\mathrm{tr}\dot{\mathbf{E}}, \quad \dot{E}_{eq} = \sqrt{\frac{2}{3}\dot{\mathbf{E}}' : \dot{\mathbf{E}}'} \quad \text{and} \quad \det\dot{\mathbf{E}}' = \frac{1}{3}\dot{E}_{ij}'\dot{E}_{ik}'\dot{E}_{jk}' \tag{5}$$

Here again, it is convenient to introduce the Lode angle related to the strain rate tensor, called η and defined by

$$\eta = \frac{1}{3}\arccos\left(\frac{4\det\dot{\mathbf{E}}'}{\dot{E}_{eq}^3}\right) \tag{6}$$

and write the strain rate deviator in the following form in its principal frame

$$\dot{\mathbf{E}}' = \dot{E}_{eq}\begin{pmatrix} \cos\eta & 0 & 0 \\ 0 & \cos(\eta - \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\eta + \frac{2\pi}{3}) \end{pmatrix} = \dot{E}_{eq}\,\mathbf{e}' \tag{7}$$

When the macroscopic dissipation is taken as a function of the three invariants $\dot{E}_m$, $\dot{E}_{eq}$, $\det\dot{\mathbf{E}}'$ or equivalently in the form $\Pi = \Pi(\dot{E}_m, \dot{E}_{eq}, \cos 3\eta)$ (note that $\cos 3\eta$ is an invariant), the macroscopic stress tensor in the porous solid is given by

$$\mathbf{\Sigma} = \frac{\partial\Pi}{\partial\dot{E}_{eq}}\frac{\partial\dot{E}_{eq}}{\partial\dot{\mathbf{E}}} + \frac{\partial\Pi}{\partial\dot{E}_m}\frac{\partial\dot{E}_m}{\partial\dot{\mathbf{E}}} + \frac{\partial\Pi}{\partial\cos 3\eta}\frac{\partial\cos 3\eta}{\partial\dot{\mathbf{E}}} \tag{8}$$