European Journal of Mechanics A/Solids 39 (2013) 50–59

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Effect of Coulomb friction on the deformation of an elastica constrained in a straight channel with clearance

Chih-Wen Liu, Jen-San Chen*

Department of Mechanical Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan

ARTICLE INFO

Article history:
Received 30 July 2012
Accepted 28 October 2012
Available online 10 November 2012

Keywords:
Friction
Constrained elastica
Sliding contact
Rolling contact

ABSTRACT

In this paper we consider the effect of friction force on the contact behavior of an elastica constrained inside a straight channel with clearance. The elastica is fed in the channel through a space-fixed input clamp and protrude out of an elastically suspended output clamp. The two pads of the output clamp are assumed to be pre-compressed. Coulomb friction is assumed to be present on the lateral walls of the channel and in the output clamp. Focus is placed on the relative motion between the input and output ends of the beam. The beam may buckle when the input end is pushed in the channel and the output end is stuck due to friction resistance. Four types of contact patterns between the elastica and the constraining channel walls may occur when the beam buckles; they are rolling point contact, combined rolling and sliding point contact, rolling line contact, and sliding line contact. When the friction resistance at the output end is overcome, the beam will protrude out of the channel. When friction is present between the elastica and the lateral walls, hysteresis occurs when the input end undergoes a load–unload cycle. A small-deformation theory is also developed for the case when the spacing between the constraining walls is small. As expected, the accuracy of the small-deformation theory deteriorates as the spacing of the constraining walls increases.

1. Introduction

Stent procedures are commonly adopted in treating patients with coronary artery diseases (Schneider, 2003). During the surgery, the surgeon inserts a guidewire into the artery of the patient through a puncture in the groin. With the help of a fluoroscopic imaging system, the guidewire is pushed to the neighborhood where artery repair is needed. In some extreme cases, the blockage of the artery is so severe that the leading end of the guidewire can only barely pass through the lesion area, as shown in Fig. 1(a). In this case the narrowed passageway may "squeeze" the guidewire and exert a friction force opposing its movement. The guidewire may then deform and contact the artery wall while it is continuously fed into the artery channel with the input end controlled by the surgeon's hand movement. As a consequence, the movement of the leading end of the guidewire near the lesion does not follow the surgeon's hand movement at the input end. This may create a problem for a surgeon in the operating room, and for a control engineer in designing a medical robot. This paper intends to investigate the deformation of the guidewire in this situation, with emphasis on the relative movement between the leading end and the input end. The guidewire is modeled as an elastica and the artery housing is modeled as a rigid channel. The lesion area is modeled as a pair of clamps suspended by torsional and translational springs, as shown in Fig. 1(b).

The elastica problem discussed in this paper falls in a field called constrained elastica, which refers to an elastica constrained laterally and forced to deform between a pair of rigid walls. Most of the previous researches in constrained elastica assume that the contact between the elastica and the rigid walls is frictionless; see Feodosyev (1977), Vaillette and Adams (1983), Adams and Benson (1986), Adan et al. (1994), Domokos et al. (1997), Holmes et al. (1999), Chai (1998), Chen and Li (2007), and Ro et al. (2010).

Effect of friction in constrained elastica was discussed previously by several researchers. Chateau and Nguyen (1991) considered the effect of Coulomb friction and formulated a rate problem leading to some stability criteria. Chai (2002) used a large-strain finite element analysis incorporating a frictional contact algorithm to model the behavior of the constrained elastica. Roman and Pocheau (2002) observed that the contact point shifting was due to either gliding, rolling, or both motions simultaneously. These previous works considered the case when the leading end of the elastica is fixed in space. This is equivalent to the case when the friction at the output end is infinite.

* Corresponding author.
E-mail address: jschen@ntu.edu.tw (J.-S. Chen).

0997-7538/$ – see front matter
http://dx.doi.org/10.1016/j.euromechsol.2012.10.011

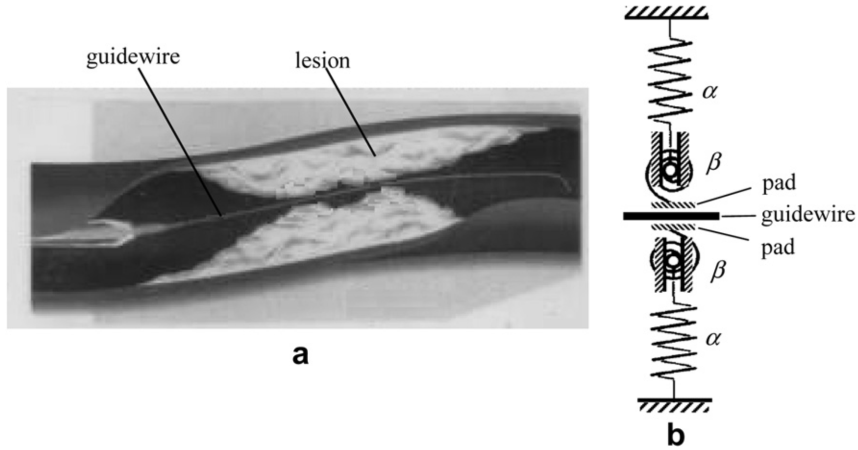

Fig. 1. (a) A guidewire passes through the lesion area of an artery channel. It is assumed that the guidewire is squeezed by the elastic tissue in the lesion. (b) The tissue in the lesion is modeled as a pair of pads suspended elastically by translational springs α and torsional springs β. The translational springs are pre-compressed by force F_1.

In contrast to these previous works, instead of being fixed in space, we consider the case when the leading end of the elastica is allowed to protrude out of the channel when the friction force at the output clamp is overcome by the pushing force. Focus will be placed on the effect of friction force at the output clamp and in the lateral contact between the elastica and the constraining walls on the movement relations between the two ends of the elastica. In Section 2, we define the problem. In Section 3, we describe several contact patterns between the elastica and the constraining walls. In Section 4, numerical results based on an elastica model are presented. In Section 5, a small-deformation theory is developed for the case when the spacing between the constraining walls is small. The results from the elastica theory and small-deformation theory are then compared. In Section 6, a comprehensive parameter study is presented. The knowledge gained in this analysis should prove useful in designing a medical robot for stent deployment in practice.

2. Problem description

Fig. 2 shows a deformed inextensible elastic beam, with flexural rigidity EI, inside a channel comprised of two straight rigid walls. The elastic beam is stress free when it is straight. The spacing of the parallel walls is $2h$. At the left end A of the channel, there is a space-fixed feeding hole. The elastic beam is fed in the channel through the hole without friction and clearance. At the right end B of the channel, there is an elastically suspended clamp. The distance between ends A and B is L. The elastic beam is guided to pass through between the upper and lower rigid pads of the clamp. The pair of pads is supported in the transverse direction by two translational springs α, and rotationally by torsional springs β. The clamp is prevented against moving in the longitudinal direction, as shown in Fig. 1(b). This clamp-spring assembly is meant to represent the stiffness of the blockage tissue inside the artery. In order to simulate the phenomenon that the guidewire be squeezed by the blockage, we assume that the translational springs are pre-compressed by force F_1. The left feeding hole and the right clamp before deformation are at the centerline of the channel.

The elastic beam is under edge thrust F_A at the left end A. When the squeezing force F_1 at the output end B is present, the pair of clamp pads exerts a Coulomb friction force against the longitudinal movement of the beam relative to the clamp. The coefficient of static friction between the beam and the clamp pad is μ_B. In the case when this friction is large enough, the beam may be stuck at end B. In this case the beam may deform inside the channel between the upper and bottom parallel walls. As a consequence, the length of the beam inside the channel increases by an amount Δl_A. Fig. 2 shows the case when the buckled beam is in point contact with the upper wall at the midpoint. When lateral contact with the plane wall occurs, it is assumed that Coulomb friction with coefficient μ_l develops. The slope of the beam at end B may become non-zero due to rotational flexibility. When the edge thrust F_A continues to increase, the beam may continue to be stuck at end B or may overcome the frictional force and protrude out of clamp B an amount Δl_B. The focus of this paper is on the effect of friction on the relation between the input length increment Δl_A and the output increment Δl_B.

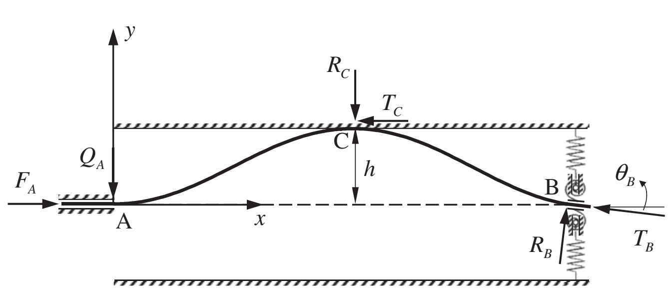

Fig. 2. A beam is fed in the channel through hole A by pushing force F_A and is stuck at clamp B due to friction resistance. The clamp B is suspended elastically. The beam is in point contact with the upper wall.

3. Contact patterns

3.1. Before contact

We assume that the friction at end B is large enough that the beam is stuck at end B and is forced to deform when the elastica is continuously fed in through end A. We consider the deformation before the beam touches the plane walls. We set an xy-coordinate system with the origin at point A, as shown in Fig. 2. Before contacting the constraining walls, the equilibrium equation at any point can be written as

$$M(s) = \mathrm{EI}\frac{\mathrm{d}\theta}{\mathrm{d}s} = M_A + F_x(s)y - F_y(s)x \tag{1}$$

s is the length of the beam measured from end A. $M(s)$ is the bending moment at point s. M_A is the bending moment exerted by