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Elastic field of a nano-film subjected to tangential surface load: Asymmetric problem

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ABSTRACT

Surface energy effects can play a significant role in the case of nanoscale materials and structures due to their high surface to volume ratio. In this paper, the general three-dimensional asymmetric problem for an elastic layer of nanoscale thickness that is bonded to a rigid substrate and subjected to tangential loading at the surface is considered. Muki's integral transforms method for classical elasticity is extended to investigate the influence of surface energy effects on the elastic field based on the Gurtin–Murdoch continuum model. Hankel integral transforms are used to solve the non-classical boundary value problems related to a tangential concentrated load and a uniformly distributed tangential circular patch load. Explicit analytical solutions are obtained for the corresponding boundary-value problems. Selected numerical results are presented to illustrate the influence of surface energy effects and layer thickness on the elastic field.

1. Introduction

Surface energy effects are known to have a significant influence on deformations and stresses in soft elastic solids and are used to explain the size-dependent behavior of nanoscale structures (Assender et al., 2002; Sharma and Wheeler, 2007; Zhang and Wang, 2007). The excess free energy associated with a surface/interface is called surface/interfacial free energy. The ratio of surface free energy γ (J/m^2) and Young's modulus E (J/m^2), γ/E, is dimensional (m) and points to some other inherent parameters of a material (Cammarata, 1994). This intrinsic length scale is usually very small for metallic materials, in the nanometer range or even smaller. When a material element has a characteristic length comparable to the intrinsic scale, the surface/interface free energy can play an important role in its properties and behavior. In the case of soft elastic solids the intrinsic length scale is not in the nanometer range but can be comparable in practical situations to the dimensions of a material specimen thus requiring the consideration of surface energy effects (He and Lim, 2006). In view of the growing interest in nanotechnology and soft materials such as biological tissues, polymer gels, etc, there is a need to examine the mechanical behavior of elastic materials in the presence of surface energy effects.

The concept of surface energy was first introduced by Gibbs (1906). Gurtin and Murdoch (1975, 1978) developed a continuum theory to account for the effects of surface and interfacial energy, in which a surface is modeled as a mathematical layer of zero thickness perfectly bonded to an underlying bulk. The surface/interface has its own properties and processes that are different from the bulk. A surface is characterized by its residual stress and elastic properties which can be determined from atomistic simulations. Shenoy (2005) conducted an atomistic calculation based on the embedded atom method (Daw and Baskes, 1984) to determine the surface properties of several different materials. It is demonstrated that size-dependent behavior of nano-scale structural elements can be modeled by using the Gurtin–Murdoch surface elasticity model (Miller and Shenoy, 2000; Shenoy, 2005). More recently, Mitrushchenkov et al. (2010) studied mechanical behaviors of wurtzite AlN nanowires via atomistic and modified continuum models with surface effects, and they showed that the continuous mechanics model gave very good accordance with the *ab initio* calculation. Jing et al. (2006) experimentally measured the surface elastic properties of silver nanowires by using three-point bending test and contact atomic force microscopy.

Several past studies have considered the response of an elastic medium in the presence of surface stresses. He and Lim (2006) derived the surface Green's functions of a soft incompressible isotropic elastic half-space including the effects of surface energy. In addition to the incompressibility, they assumed that surface elastic properties are the same as the bulk properties, which is

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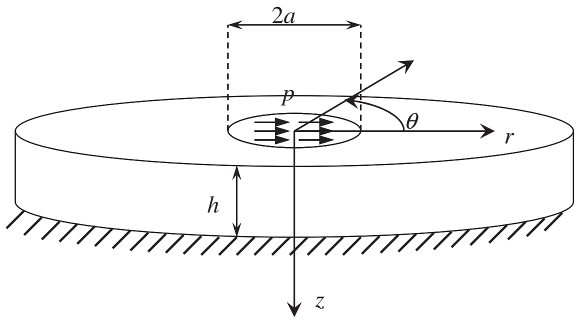

Fig. 1. Elastic layer subjected to tangential surface loading.

practically unrealistic. Wang and Feng (2007) studied the response of a half-plane subjected to surface pressures by neglecting the surface elastic constants and considering only the influence of constant surface tension. Huang and Yu (2007) considered a surface-loaded half-plane with non-zero surface elastic constants in the absence of surface tension. Zhao and Rajapakse (2009) studied the two-dimensional plane and axisymmetric surface loading problems of an elastic layer without imposing any restrictions on the surface properties. Recently, Long et al. (2012) considered a two-dimensional Hertzian contact problem between a cylinder and a half space with surface tension by neglecting the surface elasticity.

This paper presents a three-dimensional analytical solution of a nanofilm including the tangential loading in the presence of surface energy effects. Tangential loading is important in the analysis of tribology problems related to nanocoatings, nanoindentation for material characterization, cell adhesion on soft substrates, etc. (Bhushan, 2008; Lucas et al., 2004; Zhu et al., 2001). In this work, a compressible isotropic elastic layer with complete surface stress effects (non-zero surface tension and surface elastic properties) that is bonded to a rigid base and subjected to tangential surface loading is considered. The Gurtin–Murdoch model together with Hankel integral transforms are used to solve the boundary-value problems associated with the tangential loading cases. Although explicit analytical solutions can be derived for the present class of problems, closed-form solutions cannot be found. Selected numerical results are presented to demonstrate the influence of surface energy and layer thickness on the elastic field due to tangential loading.

2. General solution

An isotropic elastic layer bonded to a rigid substrate and subjected to surface tangential loading is shown in Fig. 1. The cylindrical coordinates (r, θ, z) are employed in the formulation and loading is assumed to apply over a circular area. Under these conditions, the present class of problems can be solved by using Fourier expansion in the θ-direction and Hankel integral transforms in the radial direction as shown by Muki (1960) for the classical elasticity case.

In the absence of body force, the equilibrium and constitutive equations of the isotropic materials are

$$\sigma_{ij,j} = 0 \tag{1}$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \tag{2}$$

and the classical strain–displacement relation is

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

where u_i, σ_{ij} and ε_{ij} denote the components of displacement, stress and strain tensors respectively; and μ and λ are Lamé constants of the bulk material.

Following (Muki, 1960), the general solution for the displacement and stress components in the cylindrical coordinates (r, θ, z) for an isotropic homogeneous elastic solid under loading that is symmetric about the axis $\theta = 0$ can be expressed as,

$$u_r = \frac{1}{2}\sum_{m=0}^{\infty}[U_{m+1}(r,z) - V_{m-1}(r,z)]\cos m\theta \tag{4}$$

$$u_\theta = \frac{1}{2}\sum_{m=0}^{\infty}[U_{m+1}(r,z) + V_{m-1}(r,z)]\sin m\theta \tag{5}$$

$$u_z = -\sum_{m=0}^{\infty}\int_0^{\infty}\left\{\left[A_m\xi + \left(\frac{2\mu}{\lambda+\mu} + z\xi\right)B_m\right]e^{-\xi z} + \left[C_m\xi - \left(\frac{2\mu}{\lambda+\mu} - z\xi\right)D_m\right]e^{\xi z}\right\}\xi^2 J_m(\xi r)\mathrm{d}\xi\cos m\theta \tag{6}$$

$$\frac{\sigma_{rr}}{2\mu} = \sum_{m=0}^{\infty}\left[\int_0^{\infty}\left\{\left[-A_m\xi + \left(\frac{2\lambda+\mu}{\lambda+\mu} - z\xi\right)B_m\right]e^{-\xi z} + \left[C_m\xi + \left(\frac{2\lambda+\mu}{\lambda+\mu} + z\xi\right)D_m\right]e^{\xi z}\right\}\xi^3 J_m(\xi r)\mathrm{d}\xi - \frac{(m+1)}{2r}U_{m+1} - \frac{(m-1)}{2r}V_{m-1}\right]\cos m\theta \tag{7}$$

$$\frac{\sigma_{\theta\theta}}{2\mu} = \sum_{m=0}^{\infty}\left[\frac{\lambda}{\lambda+\mu}\int_0^{\infty}\left(B_m e^{-\xi z} + D_m e^{\xi z}\right)\xi^3 J_m(\xi r)\mathrm{d}\xi + \frac{(m+1)}{2r}U_{m+1} + \frac{(m-1)}{2r}V_{m-1}\right]\cos m\theta \tag{8}$$

$$\frac{\sigma_{zz}}{2\mu} = \sum_{m=0}^{\infty}\left[\int_0^{\infty}\left\{\left[A_m\xi + \left(\frac{\mu}{\lambda+\mu} + z\xi\right)B_m\right]e^{-\xi z} - \left[C_m\xi - \left(\frac{\mu}{\lambda+\mu} - z\xi\right)D_m\right]e^{\xi z}\right\}\xi^3 J_m(\xi r)\mathrm{d}\xi\right]\cos m\theta \tag{9}$$

$$\frac{\sigma_{r\theta}}{2\mu} = \sum_{m=0}^{\infty}\left[\int_0^{\infty}\left(E_m e^{-\xi z} + F_m e^{\xi z}\right)\xi^3 J_m(\xi r)\mathrm{d}\xi - \frac{(m+1)}{2r}U_{m+1} + \frac{(m-1)}{2r}V_{m-1}\right]\sin m\theta \tag{10}$$

$$\frac{\sigma_{\theta z}}{2\mu} = \frac{1}{2}\sum_{m=0}^{\infty}(L_{m+1} + K_{m-1})\sin m\theta; \quad \frac{\sigma_{zr}}{2\mu} = \frac{1}{2}\sum_{m=0}^{\infty}(L_{m+1} - K_{m-1})\cos m\theta \tag{11}$$

and

$$U_{m+1}(r,z) = \int_0^{\infty}\left\{[-A_m\xi + (1-z\xi)B_m + 2E_m]e^{-\xi z} + [C_m\xi + (1+z\xi)D_m + 2F_m]e^{\xi z}\right\}\xi^2 J_{m+1}(\xi r)\mathrm{d}\xi \tag{12}$$