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The cylindrical interface crack in a layered tubular composite of finite thickness under torsion

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ABSTRACT

The present work investigates the problem of a cylindrical interface crack in a bi-layered tubular composite of finite thickness under torsion. Theoretical derivation is performed by the methods of Fourier integral transform and Cauchy singular integral equations. Numerical results of the stress intensity factor are discussed to reveal the coupled effects of the geometrical and physical parameters on the interfacial fracture behavior. The preferred values for the thickness ratio and the stiffness ratio are obtained, which provide necessary reference to the optimal design in engineering. To prevent interfacial fracture, it is better for the two layers to have identical thickness, and under this condition a stiffer inner layer plus a softer outer layer is a preferred choice. However, if the outer layer is enough thin, a softer inner layer plus a stiffer outer layer becomes optimal instead.

1. Introduction

Cylindrical composites are a kind of functional materials widely used in engineering. The interfaces in them are key parts but at the same time weak regions. These weak regions are generally subjected to various damages such as debonding or cracking. Therefore, fracture analysis is significant for the design and optimization of these composites, and has absorbed the attention of many researchers. When a cylindrical interface is debonded, the crack may occupy only a part of the angular range (Li and Lee, 2009, 2010a; Li et al., 2010), or even the whole angular range. Obviously, the latter case is more dangerous than the former one, and the corresponding crack is generally called a cylindrical crack.

Farris et al. (1989), Demir et al. (1992) and Zbib et al. (1995) studied the tip stress field of a cylindrical crack. Their research is important for the theory of fracture mechanics, but there is little practical significance because they assumed that the cylindrical crack occurs in a single infinite medium and this assumption can hardly hold in engineering. Comparatively speaking, it is more possible for cylindrical cracks to take place on the cylindrical interfaces of composites. Ozbek and Erdogan (1969) first studied the cylindrical cracks in fiber-reinforced composites, and their formulation keeps correct only for infinitely long cracks. Close and Zbib (1996) investigated the interaction between two cylindrical cracks in fiber-matrix composites, and for the sake of mathematical simplicity they assumed that the fiber and matrix are composed of the same material, which makes their investigation lose practical significance too. By using dual integral equations and the modified Schmidt method, Itou (1990) solved the problem of a cylindrical crack on the interface between a circular elastic cylinder and an infinite elastic medium under axial shear.

With the applications of nonhomogeneous materials, the problems of cylindrical cracks in them gradually became a hotspot. Dhaliwal et al. (1992) determined the stress intensity factor (SIF) for a cylindrical interface crack between two dissimilar nonhomogeneous coaxial finite elastic cylinders under axially symmetric longitudinal shear stress by the method of dual series equations. Han and Wang (1996), Itou and Shima (1999), Li et al. (2001) and Itou (2005) considered the cylindrical crack problems in the nonhomogeneous interfacial zone of two coaxial elastic cylinders under torsion. Their investigations would have more practical significance if the cylindrical cracks are located on the interface of the interfacial zone. Li and Weng (2001) and Feng et al. (2005) studied the cylindrical interface cracks of the nonhomogeneous interfacial zone in a composite loaded by torsional impact. It deserves noting that interfacial cracks in nine cases out of ten actually leave the interface immediately after starting to grow. Practically no cracks are found in the interface itself. Rather the crack turns to grow in one of the constituents. After such turning of the crack front the stresses are re-distributed and the stress intensity factor has to be re-calculated.

Because practical cylindrical structures are occasionally subjected to impact load, the corresponding dynamic responses of

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cylindrical crack also formed another focus in this field. Besides the papers of Li et al. (2001), Li and Weng (2001) and Feng et al. (2005), Itou (2007) studied the dynamic SIFs around a cylindrical crack in an infinite elastic medium subjected to stress waves.

All the abovementioned research concerns the problem of axisymmetric shear, which can be treated in the frame of linear elastic fracture mechanics (LEFM). However, if axial compression is applied instead, the criterion such as the SIF of LEFM will fail, because the axial compression always leads to zero SIF. In addition, when compressive loading is applied so that the crack closes, the crack will kink and continue to grow in an opening mode if the shear is large enough. Different from the abovementioned research, Guz and Khoma (2003, 2006, 2009) presented a non-classical fracture criterion based on a local stability loss to solve the problems of cylindrical cracks under compression.

Most existing papers in this field dealt with cylindrical cracks in fiber-reinforced composites with infinite substrate. However, a practical composite may only consist of a cylinder and a substrate of finite thickness. For example, two circular tubes can be bonded together to make a bi-layered tubular composite. Cylindrical interface cracks might appear in such bi-layered composites because of imperfect bonds. Therefore, fracture analysis of cylindrical interface crack is significant for the design of bi-layered tubular composites. The present paper performs such fracture analysis for a bi-layered tubular composite of finite thickness under static torsion. Fourier integral transform and the method of Cauchy singular integral equations are used to conduct the analysis. Numerical results of the SIF are provided to reveal the effects of the geometrical and physical parameters on the fracture behavior.

2. Problem formulation

Illustrated in Figs. 1 and 2 is a bi-layered tubular composite consisting of two circular tubes. The radii of the inner surface, interface and outer surface are denoted by r_0, r_1 and r_2, respectively. There is a cylindrical crack on the interface, whose length along the axis is $2a_0$. Set up a cylindrical coordinate system (r, θ, z) with the z-axis along the axis of the composite, as shown in Figs. 1 and 2. Assume that the two layers are made of two different linearly elastic materials and the composite is loaded by axi-symmetric torsion. Then the basic equations can be expressed by

$$\left.\begin{aligned}\tau_{z\theta}^{(j)} &= \mu_j \gamma_{z\theta}^{(j)} \\ \tau_{r\theta}^{(j)} &= \mu_j \gamma_{r\theta}^{(j)}\end{aligned}\right\}, \quad (j = 1,2) \tag{1}$$

$$\gamma_{r\theta}^{(j)} = \frac{\partial u_\theta^{(j)}}{\partial r} - \frac{u_\theta^{(j)}}{r}; \quad \gamma_{\theta z}^{(j)} = \frac{\partial u_\theta^{(j)}}{\partial z}, \quad (j = 1,2) \tag{2}$$

$$\frac{\partial \tau_{r\theta}^{(j)}}{\partial r} + \frac{\partial \tau_{\theta z}^{(j)}}{\partial z} + \frac{2\tau_{r\theta}^{(j)}}{r} = 0, \quad (j = 1,2) \tag{3}$$

where τ denotes the shear stress, γ the shear strain, u the displacement and μ the shear modulus. The numbers 1 and 2 in the subscripts/superscripts refer to the quantities of the inner and outer layers, respectively. By using Eqs. (1) and (2), one can rewrite Eq. (3) into

$$\frac{\partial^2 u_\theta^{(j)}}{\partial r^2} + \frac{\partial^2 u_\theta^{(j)}}{\partial z^2} + \frac{1}{r}\frac{\partial u_\theta^{(j)}}{\partial r} - \frac{u_\theta^{(j)}}{r^2} = 0, \quad (j = 1,2) \tag{4}$$

Assume that the inner and outer surfaces of the composite are loaded by torsional traction and the crack surfaces are traction free.

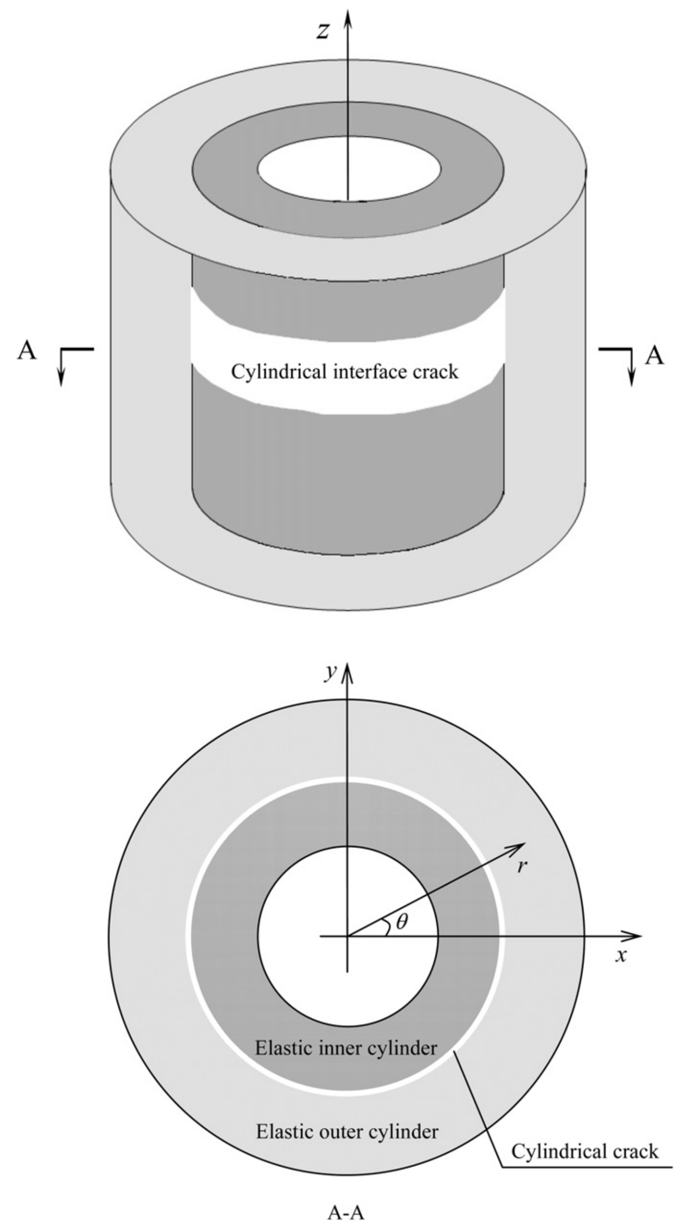

Fig. 1. A bi-layered cylinder with a cylindrical interface crack.

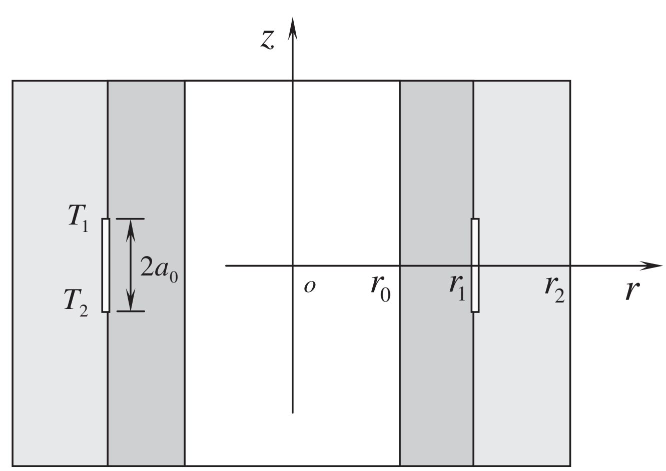

Fig. 2. The axial section of the bi-layered cylinder.