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Micromechanical modeling of piezo-magneto-thermo-elastic composite structures: Part II – Applications

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ABSTRACT

Two comprehensive micromechanical models for the analysis of piezo-magneto-thermo-elastic smart composite structures with orthotropic constituents are developed and applied to examples of practical importance. Details on the derivations of the aforementioned models are given in Part I of this work. The present paper solves the derived unit cell problems and obtains expressions for such effective coefficients as piezomagnetic, piezoelectric, elastic and many others. Of particular importance are the effective product properties, such as magnetoelectric, pyroelectric and pyromagnetic coefficients which, in general, manifest themselves in the macroscopic composite as a consequence of the interactions of the different constituents but are not exhibited by the constituents themselves as individual entities. The effective coefficients are universal in nature and once determined, can be used to examine a number of boundary value problems associated with a given composite geometry. The present work illustrates the use of the developed models and compares the results obtained with corresponding results stemming from other analytical and/or numerical models. Furthermore, results from the two micromechanical models presented here are also compared with each other. The mathematical model developed in this work can be used in analysis and design to tailor the effective elastic, piezoelectric, piezomagnetic, magnetoelectric etc. coefficients of smart composite structures to meet the design criteria of different engineering applications by a judicious selection of different geometric and/or material parameters of interest.

1. Introduction

Significant advancements in the production of composites coupled with emerging technologies in the fields of sensors and actuators have permitted the integration of smart composites in an increasingly larger number of engineering applications. Unique among smart composites are the structures made up of piezoelectric and piezomagnetic constituents. In particular, the interactions between these constituents give rise to the so-called product properties (Newnham et al., 1978) which are manifested in the macroscopic composite but are often absent from the constitutive behavior of the individual phases. Examples of such properties are the magnetoelectric, pyroelectric and pyromagnetic coefficients, see Nan et al. (2008). The magnetoelectric effect occurs via the application of a magnetic filed which causes mechanical deformation to the piezomagnetic (or magnetostrictive) constituent of the smart composite. This mechanical strain is transferred to the piezoelectric phase which in turn induces an electric displacement. Thus, overall, a magnetic field induces an electric field. The converse effect also exists; an electric field will induce a magnetic induction. Pyroelectricity refers to the development of an electric displacement through a change in temperature; similarly, pyromagnetism is manifested when a thermal expansion induces a magnetic field.

Despite the increased interest in the magnetoelectric effect and other product properties, a significant volume of research work pertaining to the micromechanical modeling of this behavior does not exist. As expected, both analytical and numerical (principally finite element-based) models have been implemented. Noteworthy among the analytical models are the works of Harshe et al. (1993), Avellaneda and Harshe (1994), Huang et al. (1996), Bichurin et al. (2003, 2010) and Ni et al. (2009). Harshe et al. (1993) and

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Avellaneda and Harshe (1994) obtained the magnetoelectric coefficients of 2-2 piezoelectric/magnetostrictive multilayer composites for mechanically free and clamped structures. Huang et al. (1996) developed a general model pertaining to piezoelectric/piezomagnetic composite materials. In a work largely reminiscent of the pioneering activities of Eshelby (1957), the authors model the inclusions as ellipsoidal reinforcements which allows the model to be tailored to a wide range of inclusion geometries including spheres, thin flakes and continuous fibers. The material tensors obtained are similar to the familiar Eshelby elastic tensors. Furthermore, employing the Mori–Tanaka Theory (1973), the authors also take into consideration the interactions between the inclusions and the matrix. Bichurin et al. (2003) investigated the magnetoelectric effect in ferromagnetic/piezoelectric multilayer composites by using an essentially averaging procedure to obtain the effective properties of the said structures. The authors employ a "classical" two-step approach in their methodology; in the first step they use the constitutive relations characterizing the individual phases and in the second step the macroscopic composite is treated as a single entity with a new set of properties which now include the product properties. The same authors, Bichurin et al. (2010), extended their work to magnetostrictive-piezoelectric nanocomposites. With respect to nanocomposites, particularly those of the 0–3 connectivity, it would not be amiss to mention at this point that special care must be exercised during any modeling approaches to take into consideration the interphase layer between the inclusions and the matrix. This is because, unlike classical composites, the interphase layer that surrounds the nanoparticles and essentially governs the transition of properties from those of the inclusion to those of the matrix is of comparable magnitude to the size of the nanoparticles themselves. Neglecting this feature may result in significant error in any modeling approach. One interesting technique to handle this issue is to first homogenize the interphase layer and obtain an effective "intermediate particle" and then apply an effective field method to the new composite. More details about this approach can be found in Sevostianov and Kachanov (2007). Ni et al. (2009) investigated the magnetoelectric properties of 3-ply polycrystalline multiferroic laminates consisting of a piezoelectric lamina sandwiched between two ferromagnetic ones. In their modeling approach the authors determine the magnetoelectric constants, α_{ij}, as the ratio of an applied magnetic field to the induced polarization. Their work shows a strong dependence of the magnetoelectric constants on the orientation of the applied magnetic fields. Akbarzadeh et al. (2011) considered, among others, the pyroelectric coefficients when analyzing the thermo-electromagnetoelastic behavior of rotating functionally graded piezoelectric cylinders subjected to thermal, mechanical and electric loading. Other works can be found in Srinivasan et al. (2001), Benveniste (1995), Nan et al. (2001), Challagulla and Georgiades (2011), and others.

Pertaining to the finite element work on piezoelectric/piezomagnetic composites, special consideration must be given to the works of Tang and Yu (2008, 2009), who employed the variational asymptotic method to investigate periodic two-phase and three-phase structures. The authors begin their approach from the total electromagnetic enthalpy or thermodynamic potential and then apply constraint minimization. The pertinent equations are solved using the finite element technique. The practically important structures considered are piezoelectric fibers embedded in a piezomagnetic matrix and piezoelectric and piezomagnetic fibers embedded in an elastic matrix. Their calculated effective coefficients conform quite well to other reported values. Sunar et al. (2002) used the finite element method to examine piezoelectric/piezomagnetic composites. The authors begin by defining two energy functionals and then apply Hamilton's principle to derive the constitutive equations (dynamic force balance, Maxwell's equations and thermal balance) pertaining to the smart structure. The authors then employ a finite element approach to a Barium Titanate/Cobalt Ferrite two-layer composite and examine the generation of a magnetic field when an applied electrostatic field induces a piezoelectric mechanical strain. The authors' results conform fairly well to those obtained via a simple analytical technique. Lee et al. (2005) developed a finite element-based model to examine, among others, the product properties developed in two-phase composites (consisting of piezoelectric fibers embedded in a piezomagnetic matrix) and three-phase composites consisting of piezoelectric and piezomagnetic fibers embedded in an elastic medium. Their results indicated that the product properties exhibited in the three-phase structure are not as pronounced as the corresponding ones induced in their two-phase counterpart because, for the materials selected, the elastic matrix is not stiff enough to effectively transfer the mechanical strain from one set of fibers to the other. Kumaravel et al. (2010) employed the finite element technique based on the Rayleigh–Ritz variational formulation to determine the critical buckling temperature and the natural frequency of three-layered and two-phase piezoelectric/piezomagnetic clamped–clamped cylinders under thermal loading. The laminated structures were made of a Cobalt Ferrite lamina sandwiched between two Barium Titanate laminae and their two-phase counterparts consisted of a cobalt ferrite matrix reinforced with Barium Titanate. In either case the cylinders were transversely isotropic with the radial direction being the axis of symmetry. Other work can be found in Liu et al. (2003, 2004), Daga et al. (2009), Mininger et al. (2010), Sun et al. (2011) and others.

An effective technique which can be used in the analysis of periodic smart composites is that of asymptotic homogenization. The mathematical framework of asymptotic homogenization can be found in Bensoussan et al. (1978), Sanchez-Palencia (1980), Bakvalov and Panasenko (1984), and Cioranescu and Donato (1999). The essence of asymptotic homogenization is centered on the fact that the micromechanical analysis of periodic composites and smart composites will inevitably lead to differential equations that are characterized by rapidly varying coefficients (with period ε, the characteristic dimension of the periodicity cell or unit cell) multiplying dependent variables which are functions of both the periodic and non-periodic variables. The non-periodic component is a manifestation of the macroscopic characteristics of the composite such as boundary conditions and external loads. The presence of the microscopic (periodic) and macroscopic scales makes an analytical solution virtually impossible in all but the simplest geometries and loading configurations; in fact, even a numerical solution is rendered computationally expensive and usually impractical for implementation in repetitive engineering design procedures. The asymptotic homogenization technique however successfully decouples the two scales so that each can be tackled independently. In most cases, the method culminates in accurate closed-form equations that can be used in design and analysis in an efficient and expedient manner.

Following the pioneering work of Duvaut (1976) who first applied the asymptotic homogenization technique to a thin plate with constant thickness and in-plane periodic inhomogeneities, Caillerie (1984), applied a modified approach to the three-dimensional heat conduction problem pertaining to a thin inhomogeneous layer. In his studies, the author employs two sets of microscopic variables; one set pertains to the tangential direction which is characterized by periodicity and the third microscopic variable pertains to the transverse direction in which there is no periodicity. A similar technique was employed by Kohn and Vogelius (1984, 1985, 1986) in their study of a thin homogeneous