Energy Conversion and Management 52 (2011) 284–291

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Direct-phase-variable model of a synchronous reluctance motor including all slot and winding harmonics

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ARTICLE INFO

Article history:
Received 6 November 2009
Accepted 29 June 2010
Available online 5 August 2010

Keywords:
Harmonics
mmf
Phase variables
Permeance
Slots
Synchronous reluctance motor
Torque pulsation
Winding function

ABSTRACT

A detailed model in direct-phase variables of a synchronous reluctance motor operating at mains voltage and frequency is presented. The model includes the stator and rotor slot openings, the actual winding layout and the reluctance rotor geometry. Hence, all mmf and permeance harmonics are taken into account. It is seen that non-negligible harmonics introduced by slots are present in the inductances computed by the winding function procedure. These harmonics are usually ignored in *d*–*q* models. The machine performance is simulated in the stator reference frame to depict the difference between this new direct-phase model including all harmonics and the conventional rotor reference frame *d*–*q* model. Saturation is included by using a polynomial fitting the variation of *d*-axis inductance with stator current obtained by finite-element software FEMAG DC®. The detailed phase-variable model can yield torque pulsations comparable to those obtained from finite elements while the *d*–*q* model cannot.

1. Introduction

The synchronous reluctance motor in its simplest form is a salient pole synchronous motor without any rotor field winding. Despite its shortcomings of lower output power and power factor, the motor is cheap, robust and capable of operating at zero slip. It is used in small sizes, where constant speed synchronous drives are required. For line-start motor operation and for good damping, a rotor cage is needed. The development of inverter drives permitted the removal of the rotor cage to give way to modifications of the rotor geometry to improve the saliency ratio on which both torque and power factor depends [1]. The presence of a cage however does not preclude the use of inverters for speed and torque control. All the benefits of the line-start version (robust rotor, damping and line-start capability) can be retained without sacrificing much of the saliency characteristics, if transverse laminated (TLA) rotors are used with the flux barriers filled with non-magnetic cage bars (aluminum or copper) as shown in Fig. 1. The design of Fig. 1 as used in this study is not optimized either for torque ripple reduction or for optimal saliency ratio.

The electrical model of both line-start and inverter fed machines (in phase variables) are differential equations having time-varying coefficients on account of the dependence of machine inductances on rotor position. Solution of such equations is challenging even for simplest cases. The *d*–*q* model was invented solely as a means of solving this problem by making the coefficients of the differential equations (inductances) constants. It turned out to have many more advantages as it revolutionalized ac machine analysis and control [2,3]. The procedure depends on the assumption that mmfs of all the windings vary sinusoidally with rotor position. The airgap permeances are modified by *Carter's* coefficients to account for slot openings. Unlike *d*–*q* models, phase-variable models can conveniently analyze machines having unsymmetrical windings, windings with different number of turns and faulty conditions [4,15]. It should be clear that a phase-variable model that does not include mmf, slot and permeance harmonics should give nearly the same results as a *d*–*q* model with the only difference being that the machine differential equations are now solved directly with its time-varying coefficients [5]. In such cases, the *d*–*q* model merely reduces the complexity of an already reduced-order model equation.

The purpose of this paper is to use the actual winding placement positions and the geometry of the airgap including both stator and rotor slots to analyze the machine and see to what extent this differs from the sinusoidally assumed *d*–*q* models. To accomplish this, we utilize a combination of the winding function theory and a direct-phase-variable model for analysis. A substantial part of the paper is devoted to modeling of the machine parts for

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doi:10.1016/j.enconman.2010.06.069

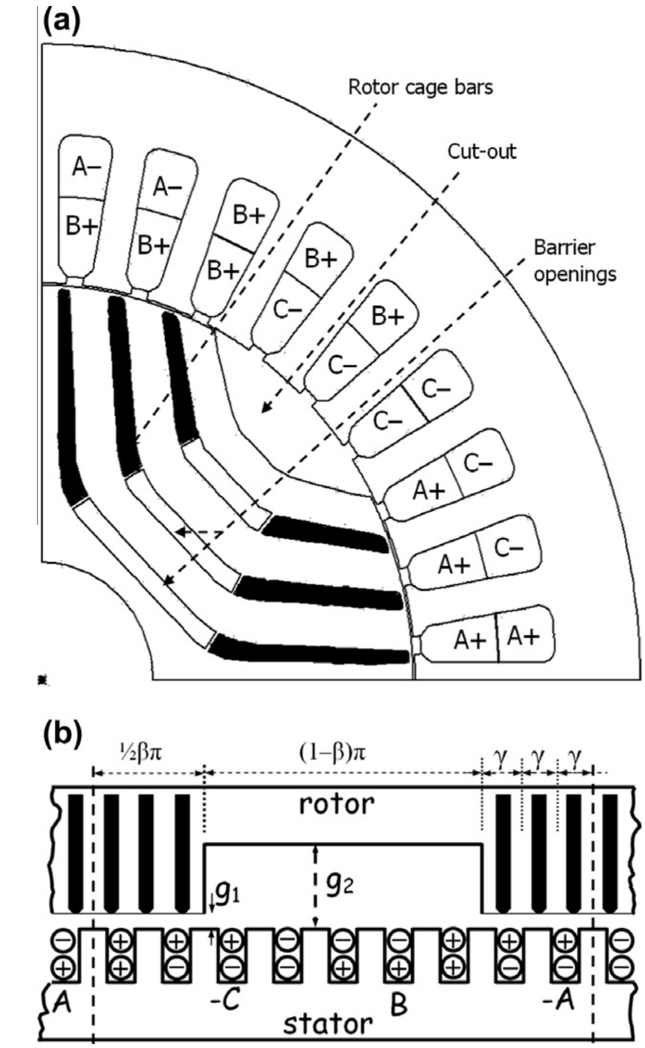

Fig. 1. (a) Axial cross-section of the synchronous reluctance machine and (b) the winding layout.

accurate calculation of inductances using direct winding function method [12].

The authors are aware of a technique known as *coupled-circuit model* [6] which considers two adjacent rotor bars as a winding loop. The method can readily yield the instantaneous rotor bar and end-ring currents. This study avoids such methods on the grounds that too many parameters are required, and because it does not bring in the effects of slot, mmf and permeance variations directly. Thus the instantaneous bar and end-ring currents are sacrificed for the rotor currents in the equivalent d- and q-axis. The advantage of the procedure reported here is that it provides a common basis for comparison of the phase-variable model with the conventional d–q model.

2. The direct-phase-variable model

Phase-variable model simulation of synchronous machines was first reported in [4] as a way of predicting machine performance during faults and unbalanced loading conditions. In [7], the effect of saturation was introduced while Abdel-Halim and Manning [8] studied the effect of additional dynamic saturation and included results of different loading conditions. A common feature of [4,7,8] is that they all neglected higher order harmonics. Working with a permanent magnet motor, Mohammed et al. [9] simulated machine performance in direct-phase co-ordinates, using time-stepping finite-element software, but did not indicate the harmonic content of the inductances although the results show its effects on machine performance. The study conducted in [10] on induction motor used an object-oriented software DYMOLA® and included the effects of slot and winding harmonics and provided results on torque pulsations.

The analysis that follows is for the machine shown in Fig. 1, which has three symmetrical phase windings in the stator. The cage is modeled as two equivalent windings, one in the q-axis and the other in the d-axis. In the stator reference frame, the first-order differential equations describing the electrical circuit of a three-phase reluctance motor using conventional notations are given as (1) while (2) are the model equations of the same motor transformed onto d–q rotor reference frame.

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \\ V_{qr} \\ V_{dr} \end{bmatrix} = \begin{bmatrix} R_{as} & 0 & 0 & 0 & 0 \\ 0 & R_{bs} & 0 & 0 & 0 \\ 0 & 0 & R_{cs} & 0 & 0 \\ 0 & 0 & 0 & R_{qr} & 0 \\ 0 & 0 & 0 & 0 & R_{dr} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{qr} \\ i_{dr} \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} L_{asas} & L_{asbs} & L_{ascs} & L_{asqr} & L_{asdr} \\ L_{bsas} & L_{bsbs} & L_{bscs} & L_{bsqr} & L_{bsdr} \\ L_{csas} & L_{csbs} & L_{cscs} & L_{csqr} & L_{csdr} \\ L_{qras} & L_{qrbs} & L_{qrcs} & L_{qrqr} & L_{qrdr} \\ L_{dras} & L_{drbs} & L_{drcs} & L_{drqr} & L_{drdr} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{qr} \\ i_{dr} \end{bmatrix} \right) \tag{1}$$

$$\frac{d}{dt}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} L_{qs} & 0 & L_{mq} & 0 \\ 0 & L_{Ds} & 0 & L_{md} \\ L_{mq} & 0 & L_{qr} & 0 \\ 0 & L_{md} & 0 & L_{dr} \end{bmatrix}^{-1} \times \left(\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{bmatrix} - \begin{bmatrix} r_s & \omega_r L_{ds} & 0 & \omega_r L_{md} \\ -\omega_r L_{qs} & r_s & -\omega_r L_{mq} & 0 \\ 0 & 0 & r_{qr} & 0 \\ 0 & 0 & 0 & r_{dr} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \right) \tag{2}$$

Accomplishing (2) from (1) is an established procedure [3] and relies on the assumption that all winding harmonics except the fundamental and all permeance harmonics, except the 2nd, are neglected. This assumption permits the use of *Park's* transformation matrix:

$$\mathbf{T}(\theta) = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{3}$$

The nature of the rotor necessitates that the stator self and mutual inductances, and the stator-to-rotor mutual inductances vary with rotor position θ_r. These expressions for the conventional procedure are well documented in the literature [3].

For ease of solution, (1) can be rewritten as:

$$[V] = [R][I] + \frac{d}{dt}([L(\theta_r)][I]) \tag{4}$$

or more conveniently as:

$$\frac{d[I]}{dt} = [L(\theta_r)]^{-1}\left([V] - \left\{[R] + \omega_r\left[\frac{dL(\theta_r)}{d\theta_r}\right][I]\right\}\right) \tag{5}$$

where

$$\omega_r = \frac{d\theta_r}{dt} \tag{6}$$

and $\mathbf{L}(\theta_r)$ is the 5×5 inductance matrix appearing in (1).