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Advantages in the torsional performances of a simplified cylindrical geometry due to transmural differential contractile properties

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ABSTRACT

A question which has not been addressed so far in the analysis of the twisting motion of the heart, relates to the existence of any advantages in energetic expenditure due to differential contractile properties across the wall of the ventricles. The question is addressed in this paper by a highly simplified analytical model of the ventricular cavity, based on a cylindrical geometry and set in the context of the linear theory of elasticity; however, the anisotropy of contraction is also taken into account. It is concluded, that when oppositely directed spirals of fibres in the internal and external layers of the cylinder keep the torsion within suitable limits, *i.e.* mimicking the biological context, a smaller energetic expenditure is related to a transmural pattern of contraction which is not uniform, and presents a larger epicardial contraction.

1. Introduction

It has recently been reported that the wringing motion of the left ventricle (LV) is a sensitive marker of the ventricular function and that heart pumping efficiency depends on the torsional performance of the heart (Sengupta et al., 2008; Shaw et al., 2008; Geyer et al., 2010). Moreover, the complex remodelling which changes the shape of the ventricle and the architecture of the fibres in the myocardium at the outset of cardiac insufficiency, may alter the twisting behaviour of the LV. Therefore, assessment of LV rotation represents an interesting approach for quantifying LV function (DeAnda et al., 1995; Grosberg and Gharib, 2009a; Helle-Valle et al., 2009). It is well-known that the twisting motion is an outcome of helical myocardial fibre orientation across the wall of the LV, which induces rotation of the apex, relative to the base (Streeter et al., 1969; Sengupta et al., 2008). Many studies have been carried out in the last thirty years to investigate the role of twist and its relation to global mechanics of the LV (Arts et al., 1979; Chadwick, 1982; Bovendeerd et al., 1992). In particular, the influence of fibre orientation on a few key parameters representative of the mechanics and energetics of the heart such as the distribution of fibre stress and strain, and ATP consumption, respectively, is analyzed (Vendelin et al., 2002).

However, a question which has not been addressed when exploring the twisting motion of the heart is whether there is any advantage in energetic expenditure due to differential contractile properties across the wall of the ventricles. This topic has received considerable attention from electrophysiologists as differential action potential durations (APDs) have been measured (Glukhov et al., 2010), which convey the potential for differential contractile properties across ventricular walls (Bueno-Orovio et al., 2008; Fink et al., 2011).

In Evangelista et al. (2011) the sensitivity of the LV torsion to contraction gradients was investigated through a refined FEM model of a human LV. In this paper, the question is addressed with reference to a highly simplified analytical model of the ventricular cavity based on a cylindrical geometry and set in the context of the linear theory of elasticity. With this model, the effect of uniform transmural contraction gradients on the torsional behaviour of the LV is investigated. Behind our analysis, is the assumption that contraction is linearly related to APD through calcium kinetics; hence, transmural gradients of APD and calcium transient kinetics (which go in the same direction, as recently shown in Glukhov et al. (2010); Lou et al. (2011)) correspond to transmural gradient of muscle contraction. The simplified modelling does not influence

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the objective, as it is shown by referring to previously published literature about the links between fibres arrangement and torsional behaviour; on the other hand, it allows the discussion of the advantages of the energetic expenditure, which might be behind the differential contractile properties across the wall of the ventricles. The results of the study agree with those in Evangelista et al. (2011) and confirm that: (**i**) consideration of the transmural gradients of contraction is fundamental in the analysis of the torsion of the LV chamber; and (**ii**) mechanical considerations do not agree with the few experimental observations performed on human hearts (Glukhov et al., 2010). Hence, the need to integrate the assumption that was made regarding APD and contractility into experimental and theoretical investigations of cardiac mechano-electric interactions in healthy and/or diseased heterogeneous myocardium.

2. A preparatory problem

The first case to be considered, was where helically oriented fibres are arranged around a cylindrical surface $\mathcal{C}_{\overline{R}}$ with radius $\overline{R}$ and height L, and form an angle α with the circumferential direction. At any point identified by the cylindrical coordinates (θ, ζ), the unit normal field is $\mathbf{n}(\theta)$, and the unit tangent field $\mathbf{f}_o$ to the helices has components (cosα, sinα), with respect to the cylindrical basis defined by the unit vectors $\mathbf{n}'(\theta) = \partial_\theta \mathbf{n}(\theta)$ and $\mathbf{e}$ spanning the circumferential and longitudinal directions, respectively. It is assumed that the fibres are generically extendable, although not elastically, and are embedded in the cylindrical surface $\mathcal{C}_{\overline{R}}$ which, in turn, may be considered as an elastic surface.

Fibres may contract, and consequently their zero-stress state may also vary in response to stimuli due to microscopic processes (*e.g.* electrical activation due to ionic conductances) which are not accounted for in this study. In the language of solid mechanics, the contraction of fibres is described as a distortion δ along the fibre direction and is usually done to take into account inelastic deformations, which induce morphological and structural changes (Rodriguez et al., 1994; Nardinocchi and Teresi, 2007; Nardinocchi et al., 2011). It is assumed that a fibre contraction of intensity δ ($\delta < 0$) induces a distortion field $\mathbf{E}_o = \delta \mathbf{f}_o \otimes \mathbf{f}_o$ on the cylinder,[1] with δ equal to the relative shortening of the fibre. The components of this distortion field in the cylindrical system are:

$$E^o_{\theta\theta} = \delta(\cos\alpha)^2,\ E^o_{\theta\zeta} = \delta\cos\alpha\,\sin\alpha,\ E^o_{\zeta\zeta} = \delta(\sin\alpha)^2, \tag{1}$$

with

$$E^o_{\theta\theta} = \mathbf{E}_o\mathbf{n}'\cdot\mathbf{n}',\ E^o_{\theta\zeta} = \mathbf{E}_o\mathbf{n}'\cdot\mathbf{e},\ E^o_{\zeta\zeta} = \mathbf{E}_o\mathbf{e}\cdot\mathbf{e}. \tag{2}$$

It is easy enough to verify that this distortion field is *compatible*, in the sense that there is a displacement field $\mathbf{u}$ on $\mathcal{C}_{\overline{R}}$ with longitudinal w, circumferential v, and radial u components such that

$$\mathrm{sym}^s\nabla_s\mathbf{u} = \mathbf{E}_o, \tag{3}$$

with

$$(\mathrm{sym}^s\nabla_s\mathbf{u})\mathbf{n}' = \frac{u}{\overline{R}}\mathbf{n}' + \frac{\partial v}{\partial\zeta}\mathbf{e},\ (\mathrm{sym}^s\nabla_s\mathbf{u})\,\mathbf{e} = \frac{\partial v}{\partial\zeta}\mathbf{n}' + \frac{\partial w}{\partial\zeta}\mathbf{e}. \tag{4}$$

Here, due to the axisymmetry of the problem, it is assumed that

$$\frac{\partial u}{\partial\theta} = \frac{\partial v}{\partial\theta} = \frac{\partial w}{\partial\theta} = 0, \tag{5}$$

so that the scalar fields w, v, and u depend on the ζ-coordinate, only.[2] Boundary conditions were permanently set in such a way that the upper end of the cylinder is fixed and the bottom end is free; the first identifies the base and the second the apex of the LV. Therefore, the displacement field $\mathbf{u}$ may be evaluated by simple integration *i.e.*

$$u(\zeta) = \overline{R}E^o_{\theta\theta},\ v(\zeta) = (\zeta - L)E^o_{\theta\zeta},\ w(\zeta) = (\zeta - L)E^o_{\zeta\zeta}, \tag{6}$$

where $\zeta = 0$ identifies the free bottom end and $\zeta = L$ the upper fixed end. The longitudinal displacement $w_o = w(0)$ at the bottom basis is $-L\ \delta\ \sin\alpha^2$. The torsional rotation Φ_o is defined as

$$\Phi_o = -\frac{v(0)}{\overline{R}}; \tag{7}$$

with this a positive rotation corresponds to a counterclockwise one when viewed from the free end (as usually assumed in clinics); we have

$$\Phi_o = \frac{L}{\overline{R}}E^o_{\theta\zeta} = \frac{L}{\overline{R}}\delta\cos\alpha\ \sin\alpha. \tag{8}$$

Fig. 1 shows the fibres arranged around the cylinder when $\alpha > 0$ (left) and $\alpha < 0$ (right). In both cases, the cylinder shortens ($w_o > 0$) as $E^o_{\zeta\zeta} < 0$ for any α. On the contrary, the two arrangements have opposite effects on the torsional rotation, *i.e.* when fibres share an angle $\alpha > 0$, the contraction induces a negative rotation of the free basis of the cylinder (clockwise, when viewed from the free basis); when fibres share an angle $\alpha < 0$, the contraction induces a positive rotation of the free basis of the cylinder. The distortion field is compatible, that is, realizable, and the corresponding elastic deformations

$$\mathbf{E}_e = \mathbf{E} - \mathbf{E}_o,\ \mathbf{E} = \mathrm{sym}\,\nabla\mathbf{u} \tag{9}$$

which measure the differences between the visible deformations $\mathbf{E}$, corresponding to the displacement field $\mathbf{u}$, and the distortions $\mathbf{E}_o$, are zero. Hence, no stress is induced and no elastic strain energy is stored in this mechanical process; the external work required to contract fibres by δ is used to change the shape of the cylinder without increasing its stored elastic energy. Compatible distortions such as $\mathbf{E}_o$ are extremely important as they allow morphological changes with no elastic energetic expenditure. However, it is worth noting that sometimes non-compatible distortions in nature are welcome, as the induced stress state may retain favourable effects on the configurational changes of the material body (Burgert et al., 2007).

3. Helically wound tubes

When the helically oriented fibres are considered arranged around a thick-walled cylinder $\mathcal{C}$ of internal and external radii R_i and R_e, respectively, things change drastically. It is assumed that at any position identified by the cylindrical coordinates (R, θ, ζ), the unit vector field $\mathbf{f}_o$ tangent to the helices has components (0, cosα, sinα) with respect to the cylindrical basis defined by the unit vectors $\{\mathbf{n}, \mathbf{n}', \mathbf{e}\}$. It is assumed that the fibres are embedded in the cylinder $\mathcal{C}$, which is assumed to be comprised of an elastic material. As our fibres are elastically nonextendable, it neglects the passive reinforcement of the material due to the presence of the fibres and

[1] It is usually assumed that a fibre contraction induces an active force in the direction of the fibre (Nash and Hunter, 2000). A different approach was introduced and detailed in Nardinocchi and Teresi (2007) and applied to the modelling of cardiac tissues in DiCarlo et al. (2009); the same approach was also followed in Grosberg and Gharib (2009b), and more recently in Ambrosi et al. (2011).

[2] Of course, this assumption is strictly related to the axisymmetry of the problem which is verified when the LV chamber is modelled as a cylinder or an ellipsoid and no longer applies when the real geometry is taken into account.