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Parametric instability of a rotor-bearing system with two breathing transverse cracks

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ABSTRACT

When the rotor rotates at a constant speed, the transverse crack opens and closes alternatively, due to gravity, and thus a "breathing effect" occurs. This variance in shaft stiffness is time-periodic, and hence a parametrically excited system is expected. The parametric excitation from the time-varying stiffness causes instability and severe vibration under certain operating conditions. Current research mostly focused on the rotor with single transverse crack. There are few studies on the multi-cracked rotor system. In fact, the interaction between the multiple parametric excitations with various phasing and amplitude, which are induced by the multiple breathing transverse cracks, would make the instability behavior of the system differ distinctly from that of the single cracked rotor system. Moreover, how the instability regions change with various crack breathing mechanisms should also be investigated. Thus, the parametric instability of a rotor-bearing system with two breathing transverse cracks is studied in the paper. First, the finite element equations of motion are established for the cracked rotor system. Two types of crack breathing mechanisms, of which one is more accurate (new) and the other is empirical (old), are adopted in the finite element formulation. Then, a generalized Bolotin's method is introduced for determining the boundaries of the primary and secondary instability regions. Based upon these, instability analysis for a practical used rotor-bearing system with single and two cracks are conducted, respectively. The instability regions induced by the single transverse crack with new and old breathing mechanisms are compared with each other. For the two-cracked rotor system, the variations of the unstable boundaries with crack depths, orientation angles and positions are observed and discussed in detail. It is shown from the results that the dynamic instability of the two-cracked rotor-bearing system indeed have some unique features that differ from that of the single cracked rotor system.

1. Introduction

The effect of the presence of the transverse crack on the dynamics of the rotor has been a focus of attention for many researchers. If undetected early, such cracks can pose a potential source of catastrophic failures. Many researchers have therefore conducted extensive investigations on the dynamics of cracked rotor over the last four decades. Early research progress could be found in (Wauer, 1990; Gasch, 1993; Dimarogonas, 1996). More recently, in depth literature reviews on the dynamical modeling and analysis of cracked rotors were published by Papadopoulos (2008) and Bachschmid et al. (2010), respectively.

When the rotor rotates at a constant speed, the transverse crack opens and closes alternatively, due to gravity, and thus a "breathing effect" occurs. This variance in shaft stiffness is time-periodic. Hence, the dynamic behavior of the rotor with breathing transverse cracks could be modeled by a coupled set of second-order linear differential equations with periodic coefficients. This type of system is often referred to as the parametrically excited system. The parametric excitation from the time-varying stiffness causes instability and severe vibration under certain operating conditions. Thus, many attentions have been paid to study the determination of operating conditions of parametric instability in cracked rotor dynamic analysis. To name a few, Meng and Gasch (2000) investigated the stability and the stability degree of a cracked Jeffcott rotor supported on different kinds of journal bearings. Gasch (2008) presented an overview stability diagram of a Laval rotor having a transverse crack. Fu et al. (2002), Dai and Chen (2007) and Chen et al. (2007), respectively, carried out nonlinear dynamic stability analysis of a rotating shaft-disk system with a transverse crack. In their model, the mass of elastic shaft, the additional displacements caused by the crack, the geometric nonlinearity of the shaft and asymmetrical viscoelastic supports were taken into account based

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upon the energy theorem and Lagrange equation. The above analysis focused on the study of simple systems with few degrees of freedom to gain a qualitative insight into the instability phenomena.

With the successful utilization of finite element models in the area of rotor dynamics, the parametric instability analysis were also extended to finite element cracked rotor-bearing systems (Sekhar and Dey, 2000; Sinou, 2007; Ricci and Pennacchi, 2009). Sekhar and Dey (2000) studied the variation of the first stability threshold limit with crack parameters and shaft internal damping. Sinou (2007) conducted the stability analysis by applying a perturbation to the nonlinear periodic solution, and analyzed the effect of crack on the first three instability regions. Ricci and Pennacchi (2009) evaluated the stability of a steam turbo generator rotor for different values of rotating speed and crack depth. In these studies, the adopted breathing model of the transverse crack was either switching (Gasch, 1976) or harmonic (Mayes and Davies, 1984), which have been proved to be approximate and rough in modeling the crack breathing behavior. More recently, the actual breathing mechanism was presented and new breathing functions of the breathing crack were introduced by Al-Shudeifat and Butcher (2011). It is shown that the new breathing functions are considerably more accurate than the previously used functions. Thus, after considering the accurate crack breathing mechanism, how the parametric instability changes would be one of the purposes of the paper.

As the literature shows, extensive efforts have been devoted to the rotor with a single transverse surface crack. When more than one crack appears in a rotor, the dynamic characteristics of the system have not gained sufficient attentions. Tsai and Wang (1997) and Sekhar (1999), respectively, employed the transfer matrix method and finite element method to analyze the natural frequencies and corresponding mode shapes of a continuous multi-cracked rotor system. The effects of both relative distance along axis and/or orientations of cracks were considered. Wu et al. (2005) investigated the coupled lateral-torsional vibration of a rotor with two transverse cracks. For a stationary shaft with two cracks, the coupled bending vibrations on the horizontal and vertical planes were discussed by Chasalevris and Papadopulos (2008). Sekhar (2008) gave a comprehensive summary on the identification methods for the multi-cracked beam and rotor systems. However, in the above analysis, the multiple cracks are all assumed to be open and the breathing behavior is not taken into account. Only Darpe et al. (2003) studied the effect of two breathing cracks on the unbalance response of a simple Jeffcott rotor, and the parametric instability was not addressed in their study. Actually, due to the multiple breathing transverse cracks, there are multiple parametric stiffness excitations in the rotor system. As the orientation, position and depth of various cracks are different, so the multiple parametric excitations might have various amplitudes, phases and

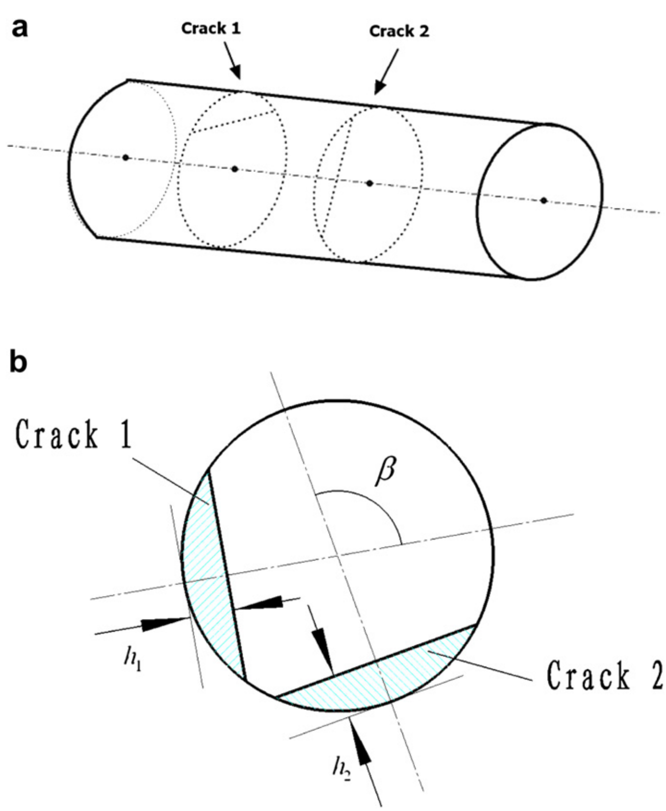

Fig. 1. Rotor having two transverse surface cracks (a) and the crack orientation in the circumference (b).

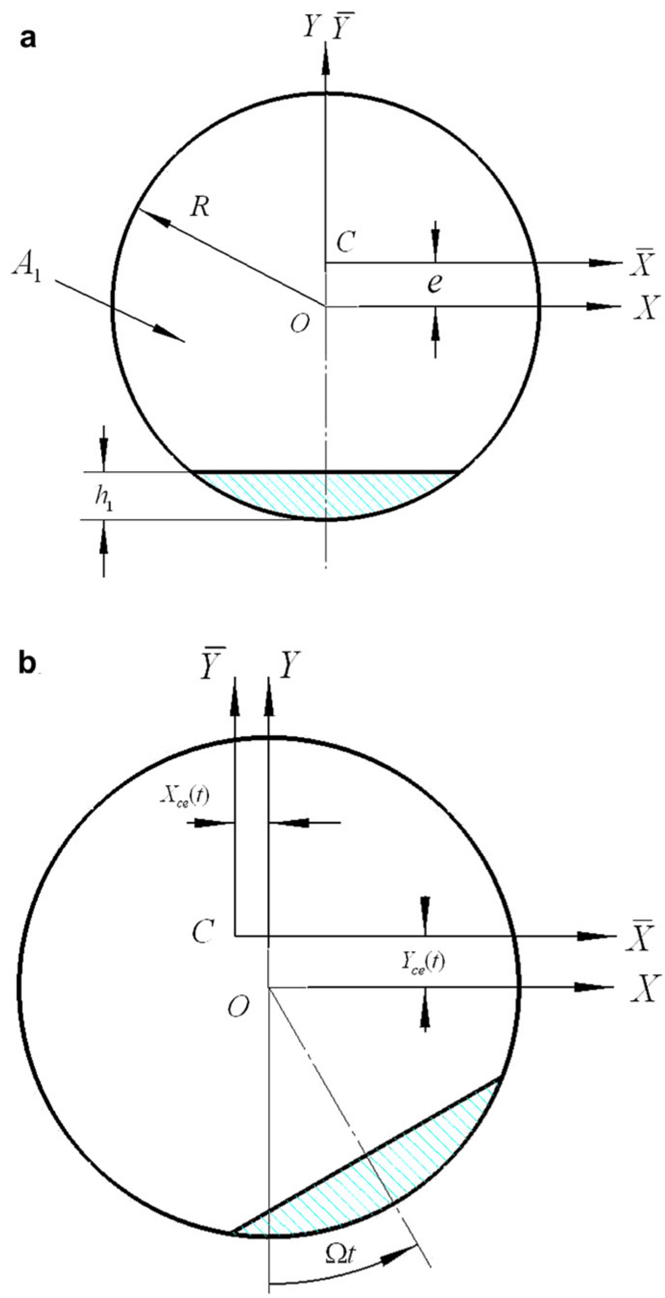

Fig. 2. Schematic diagrams of the cracked element cross-section: (a) before rotation and (b) after the shaft rotates. The dashed area represents the crack segment of crack 1.