European Journal of Mechanics A/Solids 30 (2011) 783–793

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Extension of the Linear Matching Method to frame structures made from a material that exhibits softening

O. Barrera [a], A.R.S. Ponter [b], A.C.F. Cocks [c,*]

[a] Department DASTEC, University "Mediterranea" of Reggio Calabria, Via Melissari Feo di Vito, I-89124 Reggio Calabria, Italy
[b] Department of Engineering, University of Leicester, University Road, Leicester LEI 7RH, UK
[c] Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

A R T I C L E I N F O

Article history:
Received 18 August 2010
Accepted 21 April 2011
Available online 4 May 2011

Keywords:
Maximum load
Strain softening
Linear matching method

A B S T R A C T

This paper considers the problem of evaluating the maximum load that an elastic–plastic frame structure can withstand when material or element softening is present. Here we propose an extension of the Linear Matching Method to take into account material softening. The technique has two major steps: reduction of the total potential energy to obtain the solution of a linear problem and scaling of the resulting mechanism of deformation to maximize the load. Two procedures are evaluated for the second of these steps; a direct approach which simply examines how the solution evolves along a radial path in degree of freedom space, and an incremental method which takes into account how the solution might evolve along paths away from this radial line. It is demonstrated that the incremental approach is more robust and provides stable solutions for high and low levels of softening, but numerical instabilities in the procedure can occur for intermediate degrees of softening.

1. Introduction

The maximum load that a structural component can support is determined by the softening characteristics of the material. It is well known that localized strain softening causes important consequences for the overall structural response: local buckling of beam elements in portal frames; local buckling in sandwich shell structures; and degradation in strength of composite structures due to internal cracking and fibre failure. One approach to evaluating the structural response and determining the maximum load that a structure can support is to undertake detailed incremental calculations. There is an extensive literature on the computational modelling of softening materials and numerical problems that arise as a result of loss of ellipticity of the governing incremental equilibrium equations as the deformation becomes localized (see for example Bazant, 1986 and de Borst and Mulhaus, 1992). These problems have been addressed through the introduction of rate dependence (Needleman, 1987), strain gradient (de Borst and Mulhaus, 1992) or non-local effects (Bazant et al., 1984) in the material constitutive relationships. Such calculations can be expensive to undertake and are not suitable for the early stages of design, where a wide range of different structural configurations and materials might need to be assessed.

In this paper our objective is to develop a direct method for the determination of the maximum load that a structure can support. A range of direct methods have been developed in recent years for evaluating the non-linear response of structural components which provide information that is directly relevant to the design process. The most notable examples are techniques which determine the limit state of a structure based either on upper (Ponter et al., 2000) or lower bound (Mackenzie and Boyle, 1993) approaches. Techniques have also been developed for determining the structural response under cyclic loading (Ponter and Engelhardt, 2000) and for hardening and rate dependent materials (Chen et al., 2006a,b). A major challenge is the extension of these techniques to materials that exhibit strain softening.

Different methods for modelling the response of structural systems that exhibit local softening have been proposed in the literature, see for example Cocchetti and Maier (2003), Ferris and Tin-Loi (2001) and Tangaramvong and Tin-Loi (2007,2008). An alternative approach for this class of problems is to make use of the convergence properties of the Linear Matching Method (LMM) (Ponter and Carter, 1997; Ponter and Engelhardt, 2000; Ponter et al., 2000; Ponter, 2007). Linear Matching Methods are a class of programming methods where, at each iteration, equilibrium and compatibility are satisfied and convergence is imposed by ensuring material consistency. Convergent methods have been derived for

* Corresponding author.
E-mail address: alan.cocks@eng.ox.ac.uk (A.C.F. Cocks).

0997-7538/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechsol.2011.04.009

classical limit analysis by Ponter et al. (2000) and shakedown by Ponter and Engelhardt (2000). Recently, a detailed study of convergence of both upper and lower bounds for portal frames has been carried out by Barrera et al. (2009). Here we propose an extension of the Linear Matching Method to evaluate the maximum load that portal frames with a softening moment/curvature relationship can support. Although the analyses presented in this paper relate to a particular class of frame structures and a particular type of softening behavior, the approach can potentially be applied to a wide range of structural problems and types of material behavior.

The problem and solution method described here falls within a general class of problems known as Mathematical Programming with Equilibrium Constraints (MPEC). We seek a solution where the potential energy of the structure is either minimum or stationary while, at the same time the load factor is maximized, subject to a design constraint; this is typical of MPEC problems. Such problems occur in a wide range of applications, including financial systems. Luo et al. (1996) provide a systematic mathematical treatment of such methods including a review of current solution methods. Ferris and Tin-Loi (2001), Cocchetti and Maier (2003) and Tangaramvong and Tin-Loi (2007, 2008) have used a MPEC approach for similar problems to this paper for situations where the material exhibits a small level of strain softening and, in the case of Tangaramvong and Tin-Loi (2008), second order geometric effects. Cocchetti and Maier (2003) adopt a Sequential Quadratic Programming (SQP) method where the problem is reduced to a Quadratic Programming (QP) problem, assuming that the linear hardening or softening sector of each hinge is known. A sequence of such QP problems is solved where the hardening or softening is sequentially changed for each hinge to produce an overall maximum load. Tangaramvong and Tin-Loi (2008) suggest three alternative methods for reducing the problem to a standard non-linear Programming (NP) problem which may be solved using a general purpose NP solver. They recommend a method where one of the constraints is relaxed and then sequentially tightened as the solution approaches the optimal solution. As has been observed before, the Linear Matching Method (LMM) lies outside the range of standard mathematical programming methods that have traditionally been applied to wide ranges of problems including plasticity problems; based on for example linear programming, quadratic programming and interior point methods. The LMM arises from the process of minimizing specific classes of functional that occur in non-linear plasticity and creep and does not provide a generally applicable programming method. Hence we find issues of convergence may be treated within the context of non-linear mechanics without recourse to the generalized theory discussed by Luo et al. (1996).

In this paper a frame structure is modelled using a finite element approach assuming that the elastic–plastic deformation is concentrated at "elastic–plastic hinges". The constitutive law for these hinges is expressed in terms of generalized stresses and conjugate kinematic variables (Fig. 1a). The external loads are considered monotonically increasing and proportional, so that the magnitude of the load can be described in terms of a single load factor. For this class of problem negligible "local unloading" is expected to occur and a reasonable assumption is to neglect the irreversibility by assuming path-independent constitutive models for plasticity. This assumption has been followed in previous attempts to evaluate the maximum load by Ferris and Tin-Loi (2001) and Tangaramvong and Tin-Loi (2007, 2008). For this class or model the constitutive response can be expressed in terms of a flow potential throughout the deformation process. Barrera (2010) demonstrates that for a prescribed load the total potential energy determined by the Linear Matching Method converges to the minimum possible value, but that it cannot be guaranteed that this corresponds to the exact solution when parts of the structure soften. The procedure described below generates two solutions, based on static and kinematic considerations. For the more general problem considered here we cannot guarantee that the procedure converges, but we can assess the status and accuracy of the final result by comparing the static and kinematic solutions; if they agree, compatibility, equilibrium and the constitutive response are simultaneously satisfied and the solution is exact.

The organization of the paper is as follows. In Section 2 the problem of maximization of the load is defined and a general LMM procedure is described for evaluating the maximum load. A major step in the procedure is the scaling of the solution generated by the LMM. Two approaches are explored: a direct approach, which is described in Section 3; and an incremental method, which is outlined in Section 4. Each of these approaches is applied to the analysis of frame structures and the conditions under which the methods converge and produce stable solutions are explored.

2. Maximization of the load: problem statement

The main objective of this paper is to identify a suitable procedure for the determination of the maximum load that a structure can support. In order to develop ideas, explore different methodologies, and examine the stability of these approaches and the sensitivity of the results to the detailed structure of the constitutive relationship, we concentrate on a restricted class of idealized structural problems. We examine the response of 2-D portal frames. For simplicity, we assume that deformation is concentrated

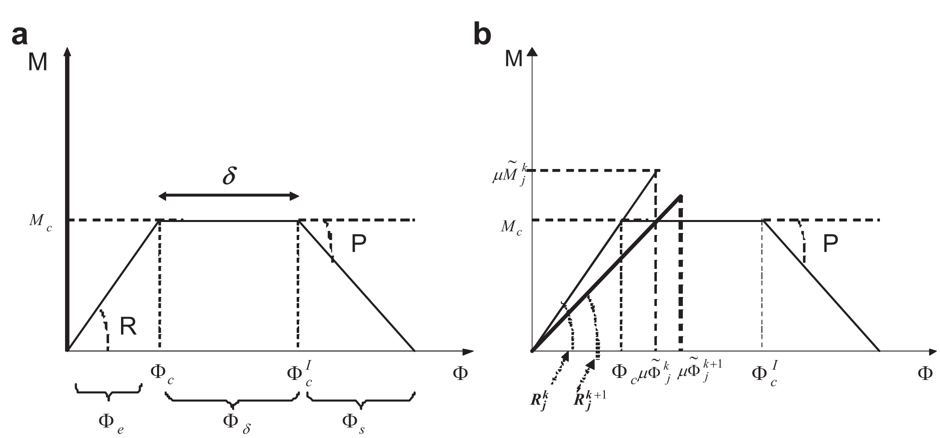

Fig. 1. (a) Material behavior. (b) Matching procedure at increment $k+1$.