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Solution of De Saint Venant flexure-torsion problem for orthotropic beam via LEM (Line Element-less Method)

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A B S T R A C T

In this paper the numerical technique, labelled *Line Element-less Method* (LEM), is employed to provide approximate solutions of the coupled flexure-torsion De Saint Venant problem for orthotropic beams having simply and multiply-connected cross-section. The analysis is accomplished with a suitable transformation of coordinates which allows to take full advantage of the theory of analytic complex functions as in the isotropic case.

A boundary value problem is formulated with respect to a novel complex potential function whose real and imaginary parts are related to the shear stress components, the orthotropic ratio and the Poisson coefficients. This potential function is analytic in all the transformed domain and then expanded in the double-ended Laurent series involving harmonic polynomials.

The solution is provided employing an element-free weak form procedure imposing that the squared net flux of the shear stress across the border is minimum with respect to the series coefficients.

Numerical implementation of the LEM results in system of linear algebraic equations involving symmetric and positive-definite matrices. All the integrals are transferred into the boundary without requiring any discretization neither in the domain nor in the contour.

The technique provides the evaluation of the shear stress field at any interior point as shown by some numerical applications worked out to illustrate the efficiency and the accuracy of the developed method to handle shear stress problems in presence of orthotropic material.
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1. Introduction

In recent years application of composite materials has found an increasing development in the construction sector. The inherent orthotropy due to their physical properties leads to mechanical behaviour characteristics that are quite different from those of conventional isotropic material requiring a proper theory of elasticity to provide stresses and strains distribution.

De Saint Venant flexure-torsion problem in prismatic beams has been widely studied from both the analytical and numerical point of view. Due to the mathematical difficulties inherent the problem, closed-form solutions exist for only a few simple cross-sectional shapes both in the isotropic and in the orthotropic case (Love, 1926; Sokolnikoff, 1956; Timoshenko and Goodier, 1970). Reference benchmark solutions in the analysis of orthotropic beams under a flexure-torsion action are represented by the series expansions provided by Lekhnitskii (1981) and Tolf (1985) while for general shape cross-sections approximate techniques must be used.

Among these techniques, powerful methods to pursue numerical solutions are mainly based on *Finite Element Method* (FEM) and *Boundary Element Method* (BEM).

A finite element approach developed on the basis of the principle of minimum potential energy was employed in Herrmann (1965) to solve torsion problem and in Mason and Herrmann (1968) to handle beams of arbitrary cross-section subjected to bending, both analyses restricted to the isotropic case. Kosmatka and Dong (1991) provided an extension of the aforementioned method applied to a discretized representation of the cross-section (Ritz method) to calculate displacement and stress distributions for an homogeneous prismatic beam of arbitrary section and rectilinear anisotropy under flexure-torsion action. The behaviour of a general anisotropic material with arbitrary cross-section was faced by using a Ritz-based power series method as reported in Kosmatka (1993). In Gruttmann et al. (1999) a finite element solution for the evaluation of the shear stresses was developed formulating all basic equations to an arbitrary coordinate system, using isoparametric element functions and introducing a stress function fulfilling the equilibrium equations.

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Both FEMs and Ritz power series method require the whole cross-section to be discretized into area elements (triangular or quadrilateral). Although the FEM approach is well behaved, in order to reach a good accuracy a large number of elements is required leading to a large set of linear algebraic equations; furthermore it is also limited with respect to the shape of the elements making the meshing process difficult; while the numerical integration in the Ritz-based methods requires a large number of integration points because of the high order of the power series.

The boundary integral methods seem to be an alternative powerful tool for the solution of the aforementioned problem requiring only boundary discretization and achieving high accuracy with a small number of boundary elements.

The boundary element procedure by using a simple constant interpolation was employed by Jawson and Ponter (1963) to solve torsion problem for homogeneous and isotropic material by Katsikadelis and Sapountzakis (1985) and Chou and Mohr (1990) to handle composite materials. The nonuniform torsion problem of composite bars of arbitrary variable cross-sections was solved by a BEM approach in Sapountzakis and Mokos (2001, 2003, 2004) and in Sapountzakis (2001). A BEM procedure with constant elements has been also used in flexure analysis for the calculation of the shear centre location by Chou (1993). A solution of a general flexure problem in a isotropic simply connected arbitrary cross-section beam was presented by Friedman and Kosmatka (2000) by means of a more refined analysis with a three-node isoparametric boundary element in order to allow an accurate geometric representation of cross-sections having curved boundaries. In the latter paper the stress field over the cross-section is described at points quite far from the boundary probably due to the difficulties implied by numerical integration.

Mokos and Sapountzakis (2005) and Sapountzakis and Mokos (2005) presented a stress function solution employing the BEM for the general transverse shear loading problem of homogeneous and composite prismatic beams of arbitrary cross-section, respectively. In Sapountzakis and Protonotariou (2008) the BEM is employed to develop a displacement solution for the general transverse shear loading problem in prismatic beams of arbitrary simply or multiply connected cross section.

Solution of De Saint Venant torsion problem obtained via FEM and BEM has also been used to evaluate torsion factors (Petrolo and Casciaro, 2004).

Within the orthotropic analysis a boundary element model in the analysis of De Saint Venant flexure-torsion problem was presented in Gaspari and Aristodemo (2005) where the differential equations governing the shear stress field are converted in three Neumann problems which form the basis of the boundary integral formulation and a quadratic B-spline approximation represents the boundary variables.

Related methods using complex variables and intending to solve boundary problems of Laplace equation distinguish in *Complex Polynomial Method* (CPM) (Hromadka and Guymon, 1984; Poler at al., 2008) and *Complex Variable Boundary Element Method* (CVBEM) (Hromadka and Lai, 1987; Hromadka and Whitley, 1998).

The application of complex variables in the boundary element methods allows to obtain the solution for two conjugate functions at once (Muskhelishvili, 1953) and both the above methods (Aleynikov and Stromov, 2004) provide solution of two-dimensional potential problems by generalization of the Cauchy integral formula into a boundary integral equation method. The CVBEM was employed also to handle nonuniform torsion with isotropic homogeneous material (Hromadka and Pardoeon, 1985) and for pure torsion of orthotropic prismatic bar (Dumir and Kumar, 1993).

Recently a boundary approach labelled *Line Element-less Method* (LEM) was introduced to solve the De Saint Venant pure torsion problem (Di Paola et al., 2008) and its extension to the flexure-torsion problem (Di Paola et al., 2011) for an isotropic material and simply and multiply connected cross-sections. Framed in the complex analysis context, LEM works in terms of a novel complex potential function related directly with the shear stresses and satisfying the field equations in the whole domain.

This complex function, holomorphic in all the domain, takes full advantage of the double-ended Laurent series involving harmonic polynomials. The Laurent series coefficients are then evaluated by employing an element-free weak-form procedure imposing that the square value of the total net flux across the border remains minimum with respect to parameters expansion under the condition that the static equivalence is satisfied. Numerical implementation of LEM results in systems of linear algebraic equations in positive-defined and symmetric matrices solving only contour integrals. The method provides exact solutions when available and high accurate results in the other cases by using few terms in the expansion. In Barone et al. (2011) it is shown how competitive is the LEM in comparison with CPM and CVBEM especially in returning exact solutions.

The LEM was already extended to the case of orthotropic material restricted to the pure torsion problem (Santoro, 2010). In this paper the LEM procedure is employed to provide solutions of the coupled flexure-torsion De Saint Venant problem for orthotropic beams having simply and multiply-connected cross-section.

The analysis of the beam is accomplished with a suitable transformation of coordinates (Sokolnikoff, 1956) and the problem is formulated with respect to a novel complex potential function, analytic in all the domain, whose real and imaginary parts are related to the shear stress components and to the parameters characterizing the orthotropy, namely the orthotropic ratio and the Poisson coefficients. Some numerical results for simply and multiply-connected domains have been reported to show the accuracy and the efficiency of the proposed method in the computation of the shear stresses in presence of orthotropic material.

2. Formulation of flexure-torsion problem for orthotropic beams

By definition, an *orthotropic* material has two or three mutually orthogonal planes of elastic symmetry, where mechanical properties are independent of direction within each plane.

Such materials require 9 independent variables (i.e. elastic constants) in their constitutive matrices. The 9 independent elastic constants in orthotropic constitutive equations are comprised of 3 Young's moduli E_x, E_y, E_z, the 3 Poisson's ratios $\nu_{zx}, \nu_{zy}, \nu_{xy}$, and the 3 shear moduli G_{zx}, G_{zy}, G_{xy}.

By choosing the axes normal to the three planes of symmetry, the compliance matrix takes the form

$$\begin{vmatrix}\epsilon_x\\ \epsilon_y\\ \epsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{vmatrix} = \begin{vmatrix} 1/E_x & -\nu_{yx}/E_y & -\nu_{zx}/E_z & 0 & 0 & 0\\ -\nu_{xy}/E_x & 1/E_y & -\nu_{zy}/E_z & 0 & 0 & 0\\ -\nu_{xz}/E_x & -\nu_{yz}/E_y & 1/E_z & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G_{yz} & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G_{xz} & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G_{xy}\end{vmatrix}\begin{vmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{vmatrix} \tag{1}$$

It's worth to be $\nu_{ij}/E_i = \nu_{ji}/E_j$ where generally $\nu_{ij} \neq \nu_{ji}$.

Assuming $\sigma_x = \sigma_y = \tau_{xy} = 0$ as in the De Saint Venant problem the constitutive equations become:

$$\epsilon_x = -\frac{\nu_{zx}}{E_z}\sigma_z; \epsilon_y = -\frac{\nu_{zy}}{E_z}\sigma_z; \epsilon_z = \frac{1}{E_z}\sigma_z; \gamma_{xy} = 0; \gamma_{zy} = \frac{\tau_{zy}}{G_{zy}}; \gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} \tag{2}$$

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