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Shear buckling of infinite plates resting on tensionless elastic foundations

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ABSTRACT

The buckling problem of an infinite thin plate resting on a tensionless Winkler foundation and subjected to shearing loads is investigated. The infinite plate is simplified to a one-dimensional mechanical model by assuming a lateral buckling mode function and a borderline function between contact and non-contact regions. After the governing differential equations for the plate sections in the contact and non-contact regions have been solved, the problem reduces to two nonlinear algebraic equations. Buckling coefficients for plates with simply supported edges and clamped edges are determined for a range of relative foundation stiffness factors. Comparison of the results with existing theory and finite element analyses shows good agreement.

1. Introduction

Laminated composite members which consist of an external steel sheet containing a light-weight filler are widely used in the civil engineering industry. Under compressive or shear stress conditions, the external steel skin is likely to exhibit a type of unilateral contact buckling in which sections of the skin become separated (delaminated) from the filler material and buckle away from it, while other sections maintain contact with the filler material. This type of buckling problem may be modelled as a thin plate (steel skin) supported by a tensionless elastic foundation (filler material), leading to a problem which is difficult to analyse due to the nonlinearities resulting from the unilateral constraint and the complexity of contact effects. One challenge during the solution procedure is the determination of unknown boundary conditions (contact zones and non-contact zones).

Existing literature relevant to the topic has focused mainly on the analysis of buckling under direct compressive loading. Seide (1958) studied a simply supported infinite plate resting on rigid foundations. Shahwan and Waas (1994) and Smith et al. (1999a) studied unilaterally constrained finite plates with different boundary conditions. Wright (1995), Uy and Bradford (1996), Smith et al. (1999b, 1999c) and Ma et al. (2008a) studied the local buckling problems in composite steel-concrete members. To consider the deformation of an elastic foundation, Chai et al. (1981) studied a one-dimensional delamination buckling problem through a beam-column model. Shahwan and Waas (1998) presented an infinite plate model for plates resting on tensionless Winkler foundations. Ma et al. (2007) numerically simulated the buckling responses of two plates in unilateral contact. Taking clamped plates for example, the relationship between elastic foundation model, rigid foundation model, infinite plate model and finite plate model was clarified by Ma et al. (2008b). Studies of post-buckling behaviour of plates supported by tensionless rigid/elastic foundations were conducted by Chai (2001), Holanda and Goncalves (2003) and Shen and Li (2004). Regarding the buckling behaviour of plates in pure shear, Smith et al. (1999d) studied finite-length plates with combinations of clamped, simply supported and free edges. Buckling responses of plates with aspect ratios varying between 1 and 4 were given. In general, a significant computational and modelling effort is required in order to achieve an accurate and stable buckling coefficient for plates with large aspect ratios. For the case of a long plate, a simplified infinite plate model is more suitable for bilateral buckling analysis (Timoshenko, 1936) and unilateral contact buckling analysi (Ma et al., 2008a, b). However, the shear buckling solution (Ma et al., 2008a) was focused on the rigid foundation case. As a further study, this paper addresses the problem of infinite plates resting on tensionless elastic foundations and endeavours to discover the relationship between shear contact buckling coefficients and the foundation stiffness.

2. Buckling analysis of infinite plate in pure shear

Ignoring end effects, a relatively long strip resting on a tensionless Winkler foundation and loaded by in-plane shearing

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stresses reduces to an infinite thin plate model with periodically repeating buckles as shown in Fig. 1.

Considering a single wave length plate section, the governing equation for an isolated thin plate may be expressed as

$$w_{i,x_ix_ix_ix_i}+2w_{i,x_ix_iyy}+w_{i,yyyy}-\frac{2\pi^2K}{b^2}w_{i,x_iy}$$
$$=-\frac{k_iw_i}{D}\quad |x_i|\le a_i/2\quad i=1,2 \tag{1}$$

where $D=\mathrm{Et}^3/12(1-\nu^2)$; $K=b^2\tau t/\pi^2D$; $k_i=0$: non–contact area/k : contact area; a_i is half-wave-length of the ith plate section; b and t are plate width and thickness; D, E, ν are flexural rigidity, elastic modulus and Poisson's ratio of the plate;τ is shear stress; w_i is vertical displacement in the ith plate section;x_i is local longitudinal coordinate in the ith plate section; y is transverse coordinate; k is stiffness factor of the Winkler foundation; the subscript, $x_i(y)$ indicates partial differentiation $\partial/\partial x_i(\partial/\partial y)$, etc, the subscript $_i$ indicates that the parameter relates to plate section i ($i=1$ for non-contact plate section, $i=2$ for contact plate section).

For buckling analysis of an infinite strip, an approximate deflection surface function expressed through the combination of the lateral mode, longitudinal mode and the zero deflection nodal line may be assumed as

$$w_i(x_i,y)=f_i(\bar{x}_i)g(y) \tag{2}$$

where $f_i(\bar{x}_i), g(y)$ are longitudinal and lateral buckling mode functions respectively; $\bar{x}_i=x_i-s(y)$; $s(y)$ is the borderline function defining the border between contact and non-contact regions. For non-dimensional analysis, Eq. (2) may be rewritten as

$$w_i(x,y)=\tilde{w}_i(\xi_i,\eta)=\tilde{f}_i(\xi_i)\tilde{g}(\eta) \tag{3}$$

where buckling mode functions $\tilde{f}_i(\xi_i)=f_i(\bar{x}_i), \tilde{g}(\eta)=g(y)$, non-dimensional parameters $\xi_i=\bar{x}_i/a_i=[x_i-s(\eta)]/a_i$, $\eta=y/b$. The lateral buckling mode function $\tilde{g}(\eta)$ may be assumed as (Ma et al., 2008a)

$$\tilde{g}(\eta)=\cos\ \pi\eta\quad \text{for a simply supported plate} \tag{4a}$$

and

$$\tilde{g}(\eta)=\left[1/4-\eta^2\right]^2\quad \text{for a clamped plate.} \tag{4b}$$

The borderline function may be assumed as

$$s(y)=s(\eta)=b\left[c_1\eta+c_3\eta^3+c_5\eta^5+\cdots+c_N\eta^N\right] \tag{4c}$$

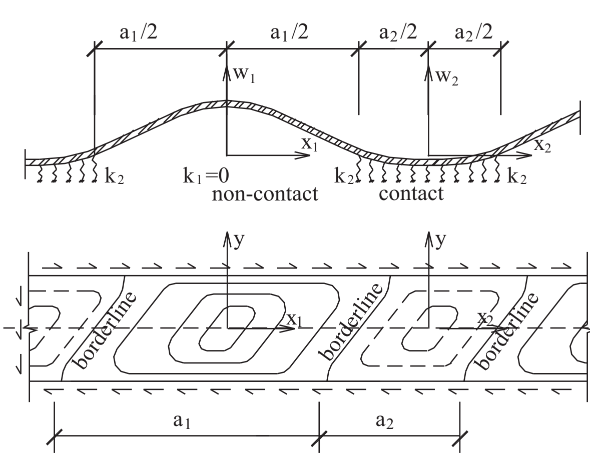

Fig. 1. Buckling mode of an infinite plate in pure shear.

where N is order of the highest order y term in borderline polynomial. For simply supported plates, the boundary conditions may be expressed as

$$\tilde{w}_i(\xi_i,\eta)=\tilde{f}_i\tilde{g}=0\quad \eta=\pm 1/2 \tag{5a}$$

$$\tilde{w}_{i,\eta\eta}(\xi_i,\eta)=\tilde{f}_i''s'^2\tilde{g}-\tilde{f}_i's''\tilde{g}-2\tilde{f}_i's'\tilde{g}'+\tilde{f}_i\tilde{g}''=0\quad \eta=\pm b/2 \tag{5b}$$

For clamped plates, the boundary conditions are

$$\tilde{w}_i(\xi_i,\eta)=\tilde{f}_i\tilde{g}=0\quad \eta=\pm 1/2 \tag{5c}$$

$$\tilde{w}_{i,\eta}(\xi_i,\eta)=-\tilde{f}_i's'\tilde{g}+\tilde{f}_i\tilde{g}'=0\quad \eta=\pm 1/2 \tag{5d}$$

where primes denote differentiation with respect to ξ_i or η.

For clamped plates, the assumed buckling mode based on (4b) exactly satisfies the boundary conditions (5c) and (5d). However, for simply supported plates, the assumed buckling mode (4a) does not automatically satisfy the boundary condition of (5b). Thus we require an additional equation for simply supported plates

$$s'(\eta=\pm 1/2)=c_1+3c_3(1/2)^2+5c_5(1/2)^4+\cdots+Nc_N(1/2)^{N-1}$$
$$=0\quad N\ge 3 \tag{6}$$

Substituting Eq. (3) into Eq. (1) and integrating (1) after multiplying both sides by function $\tilde{g}(\eta)$, we have

$$\tilde{f}_i''''-B_1\gamma_i^2\tilde{f}_i''+B_2\gamma_i^4\left(1+\tilde{k}_i\right)\tilde{f}_i=0 \tag{7}$$

where

$$B_1=\int_{-1/2}^{1/2}[2(1+3s'^2)\tilde{g}''+(4s's'''+3s''^2)\tilde{g}+12s's''\tilde{g}'$$

$$2s'\pi^2K\tilde{g}]\tilde{g}d\eta/\int_{-1/2}^{1/2}(1+2s'^2+s'^4)\tilde{g}^2d\eta$$

and

$$B_2=\int_{-1/2}^{1/2}\tilde{g}''''\tilde{g}d\eta/\int_{-1/2}^{1/2}(1+2s'^2+s'^4)\tilde{g}^2d\eta.$$

The symmetric solution of (7) may be written as

$$\tilde{f}_i(\xi_i)=A_{1i}f_{1i}+A_{2i}f_{2i} \tag{8}$$

where the functions f_{1i} and f_{2i} depend on the value of the parameter $\Delta_i=B_1^2/4-B_2(1+k_i)$ as follows:

Case 1. $\Delta_i>0$

$$f_{1i}(\xi_i)=\cos\ (\alpha_i\xi_i) \tag{9a}$$

$$f_{2i}(\xi_i)=\cos\ (\beta_i\xi_i) \tag{9b}$$

$$\alpha_i,\beta_i=\gamma_i\left[-B_1/2\pm\sqrt{\Delta_i}\right]^{1/2} \tag{9c}$$

Case 2. : $\Delta_i=0$

$$f_{1i}(\xi_i)=\cos\ (\alpha_i\xi_i) \tag{10a}$$

$$f_{2i}(\xi_i)=\xi_i\sin\ (\alpha_i\xi_i) \tag{10b}$$

$$\alpha_i=\gamma_i[-B_1/2]^{1/2} \tag{10c}$$

Case 3. $\Delta_i<0$

$$f_{1i}(\xi_i)=\left(e^{\alpha_i\xi_i}+e^{-\alpha_i\xi_i}\right)\cos(\beta_i\xi_i) \tag{11a}$$

$$f_{2i}(\xi_i)=\left(e^{\alpha_i\xi_i}-e^{-\alpha_i\xi_i}\right)\sin\ (\beta_i\xi_i) \tag{11b}$$