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Statistical approach for a hyper-visco-plastic model for filled rubber: Experimental characterization and numerical modeling

J.M. Martinez [a], A. Boukamel [a,*], S. Méo [b], S. Lejeunes [a]

[a] Laboratoire de Mécanique et d'Acoustique, CNRS UPR-7051, 31 chemin Joseph-Aiguier, 13402 Cedex 20, Marseille, France
[b] Laboratoire de Mécanique et Rhéologie, EPU Dept Productique, BP 407, F37204 Cedex 3, Tours, France

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ABSTRACT

This paper presents a campaign of experimental tests performed on a silicone elastomer filled with silica particles. These tests were conducted under controlled temperatures (ranging from −55 °C to +70 °C) and under uniaxial tension and in shearing modes. In these two classes of tests, the specimens were subjected to cyclic loading at various deformation rates and amplitudes and relaxation tests at various levels of deformation. A statistical hyper-visco-elasto-plastic model is then presented, which covers a wide loading frequency spectrum and requires indentifying only a few characteristic parameters. The method used to identify these parameters consists in performing several successive partial identifications with a view to reducing the coupling effects between the parameters. Lastly, comparisons between modeling predictions and the experimental data recorded under harmonic loading, confirm the accuracy of the model in a relatively wide frequency range and a large range of deformations.

1. Introduction

Elastomers belong to the high polymer family *i.e.* they consist of macromolecular chains of various lengths, with and without ramifications. This structure confers on these materials a low level of rigidity and a high level of deformability. In addition, the reinforcement of these materials with fillers accentuates their dissipative behavior. Because of these properties, especially their damping capacity, these materials are widely used in industry. The application on which this study focuses is that of the drag dampers for helicopters. These parts connect helicopter blades to the rotor and attenuate the drag movement. Designing these parts, which are often related to safety, imposes a guarantee of high reliability under extreme operating conditions (dynamic loading with multi-frequency and large amplitudes, thermal constraints, etc). Meeting these specifications requires good knowledge of the mechanical behavior of the constitutive materials. In addition, the behavior of an elastomer can depend heavily on the temperature, the degree of cross-linking and the type of particles incorporated (carbon or silica), etc.

During recent decades, several approaches have been used by previous authors to model various behavioral aspects of elastomers:

- To describe the static behavior of the material, a hyperelastic approach was used in: Treloar (1943, 1957) where statistical models were proposed; and Mooney (1940); Rivlin (1958); Hart-Smith (1966); Ogden (1972), which involved the use of phenomenological approaches.
- Some authors have used the damage mechanics approach to describe the softening behavior occurring during the first loading cycles, which is known as the Mullins effect, see Mullins (1947); Harwood et al. (1967). A theoretical framework was proposed by Govindjee and Simo (1991, 1992) in the case of a hyperelastic behavior. A similar approach was described in Simo (1987); Miehe (1995) in the case of a visco-elastic material.
- In Holzapfel and Reiter (1995); Holzapfel and Simo (1996a,b); Lion (1997), a thermomechanical coupling model was developed, which takes into account the temperature dependence of the mechanical characteristics and describes the temperature changes resulting from the mechanical dissipation.
- Furthermore, to model the visco-elastic effects of these materials, a framework of the Finite Non-Linear Visco-elasticity has been proposed by several authors. These models can be classified in the following groups, depending on the type of formulation used:

Those using an integral approach, which was mainly developed for modeling non-linear materials with evanescent memory. These approaches describe the behavior of the material using equations

* Corresponding author.
E-mail address: adnane.boukamel@ec-marseille.fr (A. Boukamel).
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giving the stress tensor in terms of the strain history. Rivlin (1958); Coleman (1964); Christensen (1971); Coleman and Noll (1961); Lianis (1963); Chang et al. (1978); Morman (1988).

Those using a differential approach, based on the concept of intermediate states commonly used to describe finite elastic–-plastic deformations (see Sidoroff (1973, 1974)). Defining intermediate states provides the internal variables needed to describe the behavior. This approach can be said to be an extension of rheological models in the case of large strains: Sidoroff (1977); Le Tallec (1990); Le Tallec and Rahier (1994); Leonov (1992). The local state method, Lemaître and Chaboche (1996), provides the theoretical framework of this formulation, and the internal variables are provided by the intermediate states.

And those using micro-physically motivated models for filled elastomers, which are often based on hypotheses about the interactions between the agglomerates of fillers and the gum matrix: Drozdov (2001a,b; Drozdov and Dorfmann (2002, 2003); Drozdov et al. (2004)), or about the mechanisms underlying the deformation and rearrangement of the macromolecular network: Tanaka and Edwards (1992); Drozdov (1998, 2000); Reese (2003).

In this paper, a meso-physically motivated approach is used to model the response of the material, in large strain and at various frequencies and temperatures. A statistical approach is then proposed to develop a model based on the generalization of an assembly of rheological models. The advantage of this statistical rheological model is that it can be used to simulate the behavior of the material in a wide frequency range while requiring only a few parameters to be identified.

First we present the results of a series of experimental tests, which were carried out on a silicone elastomer filled with silica. These tests were uniaxial tension and shear tests and were performed under controlled temperature (ranging from -55 °C to $+70$ °C) and under various loading conditions (Relaxation tests, quasi-static and dynamic loading at various strain rates). The results show the dependence of the behavior of the material on the temperature, as well as on the strain rate (Fletcher-Gent effect, see Fletcher and Gent (1953)) and the amplitude of the strain (Payne effect, see Harwood et al. (1967)). The constitutive model is then developed on the basis of the fundamental principle of thermodynamics of continuous media, adapted to finite strain theory. Using the concept of intermediate configurations (multiplicative decomposition of the deformation gradient) and in line with the theory of thermodynamics of irreversible processes, and under the hypothesis of the normal dissipation depending only on the internal variables, the constitutive equation and the flow rules are obtained. A statistical approach is then applied, in order to extend this rheological model to a wide range of strain rates and to account for the plastic behavior of the material. In the following section, this statistical hyper-visco-plastic model is analyzed in the case of simple loads, with a view to propose a strategy for identifying its parameters. For this purpose, analytical solutions are developed to simulate the relaxation response and the hardening test, respectively. In the case of cyclic loading, a semi-analytical response was obtained using a symbolic and numeric computation software. These identifications were performed at various temperatures. Lastly, using the semi-analytical solution under sinusoidal shear loading conditions at various frequencies and amplitudes, the effect of parameters such as the temperature, frequency and loading amplitude on the harmonic response of the elastomer are analyzed.

2. Experimental analysis

2.1. Description of the experimental tests

An experimental campaign was conducted on a silicone (dimethyl-vinyl-siloxan vulcanized by peroxide) reinforced with silica particles. The glass transition temperature of this elastomer is approximately -105 °C. The following tests were carried out:

- Uniaxial tensile tests on specimens with a dumbbell shape (H2 according to standard NF T46-002), to determine the quasi-static behavior and the relaxation response of the material.
- Shear tests on Double-Shearing specimens (DS, see Fig. 1). These specimens were successively subjected to: a quasi-static loading-unloading cycle; relaxation tests at various shearing levels; triangular cyclic loading, at various strain rates (from 0.03 s^{-1}, 0.03 s^{-1} to 10 s^{-1}) and various amplitudes (12.5%, 25% and 50%).

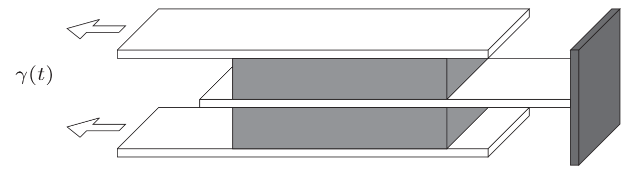

Fig. 1. Double-shearing specimen.

All these tests were performed under controlled temperatures (ranging from -55 °C to $+70$ °C) in a climatic chamber cooled by injecting nitrogen and heated with an electrical resistance and the airflow. In the tensile tests, monitoring and deformation measurements were performed with a laser extensometer.

Remark 1 (*Mullins effect*) To eliminate the Mullins effect (see Mullins (1947); Harwood et al. (1967)) and therefore to characterize the behavior of the stabilized material, a softening process was first induced by applying about ten cycles with an amplitude greater than the maximum strain imposed during the series of tests.

Remark 2 (*Temperature stabilization*) To avoid errors in the temperature measurement, a waiting period of 10 min was fixed between each characterization test to allow the temperature to reach equilibrium inside the specimen. The characterization time was sufficiently short to avoid a too strong self-heating phenomena in the specimen.

2.2. Experimental results

Relaxation tests: In the relaxation tests, the specimen was subjected to various strain levels: 25%, 50% and 100% under tension loading; 20%, 30% and 54% under shear strain. The response of the material is described by the evolution of the normalized stress[1] versus time. The curves presented in Fig. 2(a) and Fig. 2(c) show that at temperatures above the ambient temperature, the evolution of these stresses during relaxation was always independent of the strain amplitude. At these temperatures, the relaxation mechanism seems independent of the strain level, under both tension and shear loading; whereas at lower temperatures, the responses doesn't show the same linearity of the stress depending on the deformation, especially in the case of uniaxial tension tests (see Fig. 2(b)). The graphs in Fig. 2(b) and Fig. 2(d) show the dependence of the relaxation response on the temperature. The relaxation response was therefore more sensitive to the temperature in the [-55 25 °C to -25 °C] range than at higher temperatures (above 25 °C).

Quasi-static shear response: The quasi-static test was a loading-unloading test, performed at low strain rate ($\dot{\gamma} = 0.03\,s^{-1}$) and for three shear amplitudes (γ_{max} = 12.5%, 25% and 50%). The stress-strain curves given in Fig. 3 show that even at low rates of

[1] For relaxation tests, normalized stress is obtained by dividing the total stress by the instantaneous stress.