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Scheduling of head-dependent cascaded hydro systems: Mixed-integer quadratic programming approach

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ABSTRACT

This paper is on the problem of short-term hydro scheduling, particularly concerning head-dependent cascaded hydro systems. We propose a novel mixed-integer quadratic programming approach, considering not only head-dependency, but also discontinuous operating regions and discharge ramping constraints. Thus, an enhanced short-term hydro scheduling is provided due to the more realistic modeling presented in this paper. Numerical results from two case studies, based on Portuguese cascaded hydro systems, illustrate the proficiency of the proposed approach.

1. Introduction

In this paper, the short-term hydro scheduling (STHS) problem of head-dependent cascaded hydro systems is considered. In hydro plants with a large storage capacity available, head variation has negligible influence on operating efficiency in the short-term [1]. In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, operating efficiency is sensitive to the head:head change effect [2]. For instance, in the Portuguese system there are several cascaded hydro systems formed by several but small reservoirs. Hence, it is necessary to consider head-dependency on STHS. In a cascaded hydraulic configuration, where hydro plants can be connected in both series and parallel, the release of an upstream plant contributes to the inflow of the next downstream plants, implying spatial–temporal coupling among reservoirs. Head-dependency coupled with the cascaded hydraulic configuration augments the problem complexity and dimension.

Hydro plants particularly run-of-the-river hydro plants are considered to provide an environmentally friendly energy option, while fossil-fuelled plants are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option [3]. However, the rising demand for electricity, likely increases in fossil-fuel prices, and the need for clean emission-free generation sources, are trends in favor of increasing generation from renewable sources.

The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the Portuguese hydro resources reduces the reliance on fossil fuels and decreases greenhouse emissions.

In a deregulated profit-based environment, such as the Norwegian case [4] or concerning Portugal and Spain given the Iberian Electricity Market, a hydroelectric utility is usually an entity owning generation resources and participating in the electricity market with the ultimate goal of maximizing profits, without concern of the system, unless there is an incentive for it [5].

The optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for the hydroelectric utilities to face competition [6]. STHS models provide decision support for the operational task of bidding in the energy and system services markets [7].

In the STHS problem a time horizon of 1–7 days is considered, usually divided in hourly intervals. Hence, the STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities, such as inflows to reservoirs or electricity prices, the corresponding forecasts are used [8].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [9,10]. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with cascaded reservoirs is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [11].

Artificial intelligence techniques have also been applied to the STHS problem [12–15]. However, a significant computational effort

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is necessary to solve the problem for cascaded hydro systems. Also, due to the heuristics used in the search process only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded reservoirs [16]. This network flow model is often simplified to a linear or piecewise linear one [17]. Linear programming (LP) is a well-known optimization method and standard software can be found commercially. Mixed-integer linear programming (MILP) is becoming often used for STHS [18–21], where integer variables allow modeling of discrete hydro unit-commitment constraints.

However, LP typically considers that hydroelectric power generation is linearly dependent on water discharge, thus ignoring head-dependency to avoid nonlinearities. The discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve the STHS problem. Furthermore, methods based on successive linearization in an iterative scheme depend on the expertise of the operator to properly calibrate the parameters. For instance, the selection of the best under-relaxation factor in [21] is empiric and case-dependent, rendering some ambiguity to these methods.

A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric power generation characteristics more accurately and head-dependency on STHS can be taken into account [2,6,22]. Although there were considerable computational difficulties in the past to directly use nonlinear programming (NLP) methods to this sort of problem, with the drastic advancement in computing power and the development of more effective nonlinear solvers in recent years this disadvantage seems to be eliminated.

In earlier works [2,6,22], the use of the nonlinear model in some case studies leads to a result that exceeds by at least 3% what is obtained by a linear model, requiring a negligible extra computation time. However, the nonlinear model cannot avoid water discharges at forbidden areas, and may give schedules unacceptable from an operation point of view. Moreover, it is important to notice that a minor change in the electricity price may give a significant change in the water discharge, and consequently in the power generation of plants. Therefore, ramp rate of water discharge is included in the constraints to keep a lesser and steady head variation, which is particularly important for reservoirs with a task of navigation.

Hence, in this paper we propose a novel mixed-integer quadratic programming (MIQP) approach to solve the STHS problem, where integer variables are used to model the on–off behavior of the hydro plants. The proposed approach considers head-dependency, discontinuous operating regions, and discharge ramping constraints, in order to obtain more realistic and feasible results.

This paper is organized as follows. In Section 2, the mathematical formulation of the STHS problem is provided. Section 3 presents the proposed MIQP approach to solve the STHS problem. In Section 4, the proposed MIQP approach is applied on two case studies, based on Portuguese cascaded hydro systems, to demonstrate its effectiveness. Finally, concluding remarks are given in Section 5.

2. Problem formulation

The notation used throughout the paper is stated as follows.

I,i	set and index of reservoirs
K,k	set and index of hours in the time horizon
π_k	forecasted electricity price in hour k
p_{ik}	power generation of plant i in hour k
Ψ_i	future value of the water stored in reservoir i
v_{ik}	water storage of reservoir i at end of hour k
a_{ik}	inflow to reservoir i in hour k
M_i	set of upstream reservoirs of plant i
q_{ik}	water discharge of plant i in hour k
s_{ik}	water spillage by reservoir i in hour k
η_{ik}	power efficiency of plant i in hour k
h_{ik}	head of plant i in hour k
l_{ik}	water level in reservoir i in hour k
$v_i^{\min}, v_i^{\max}$	water storage limits of reservoir i
$q_i^{\min}, q_i^{\max}$	water discharge limits of plant i
u_{ik}	commitment decision of plant i in hour k
R_i	discharge ramping limit of plant i
$\boldsymbol{H}$	Hessian matrix
$\boldsymbol{f}$	vector of coefficients for the linear term
$\boldsymbol{x}$	vector of decision variables
$\boldsymbol{A}$	constraint matrix
$\boldsymbol{b}^{\min}, \boldsymbol{b}^{\max}$	lower and upper bound vectors on constraints
$\boldsymbol{x}^{\min}, \boldsymbol{x}^{\max}$	lower and upper bound vectors on variables
$\eta_i^{\min}, \eta_i^{\max}$	power efficiency limits of plant i
$h_i^{\min}, h_i^{\max}$	head limits of plant i
$l_i^{\min}, l_i^{\max}$	water level limits of reservoir i

The STHS problem can be stated as to find out the periodic water discharges, q_{ik}, the water storages, v_{ik}, and the water spillages, s_{ik}, for each reservoir, $i = 1, \ldots, I$, at all hours of the time horizon, $k = 1, \ldots, K$, hat optimize an objective function subject to constraints. The water storages at the end of the time horizon, v_{iK}, must be decided according with future operations. Additionally, the commitment decision, u_{ik}, is ascertained.

In the STHS problem under consideration, the objective function is a measure of the profit attained by the conversion of potential energy into electric energy, without affecting future operations. Thus, the objective function to be maximized can be expressed as:

$$F = \sum_{i=1}^{I} \sum_{k=1}^{K} \pi_k p_{ik} + \sum_{i=1}^{I} \Psi_i(v_{iK}) \tag{1}$$

The objective function in (1) is composed of two terms. The first term represents the profit with the hydro system during the short-term time horizon, where π_k is the forecasted electricity price in hour k and p_{ik} is the power generation of plant i in hour k.

The second term expresses the value of the water stored in the reservoirs for future operations. This second term is only needed if no final water storage requirement is specified. An appropriate representation when this term is explicitly taken into account can be seen for instance in [23]. The storage targets for the short-term time horizon can be established by medium-term planning studies.

The optimal value of the objective function is determined subject to constraints of two kinds: equality constraints and inequality constraints or simple bounds on the variables. The constraints are indicated as follows:

$$v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{mk} + s_{mk}) - q_{ik} - s_{ik} \tag{2}$$

$$p_{ik} = q_{ik}\eta_{ik}(h_{ik}) \tag{3}$$

$$h_{ik} = l_{f(i)k}(v_{f(i)k}) - l_{t(i)k}(v_{t(i)k}) \tag{4}$$

$$v_i^{\min} \leqslant v_{ik} \leqslant v_i^{\max} \tag{5}$$

$$u_{ik} q_i^{\min} \leqslant q_{ik} \leqslant u_{ik} q_i^{\max} \tag{6}$$

$$q_{ik} - R_i \leqslant q_{i,k+1} \leqslant q_{ik} + R_i \tag{7}$$