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A reliable analytical method applied to heat transfer problems associated with insulated cylindrical tanks

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Abstract

In this study, the heat transfer characteristics of insulated cylindrical tanks are analyzed by using a numerical method and three one dimensional analytical methods, namely the RPSWT (Regular Polygon top Wedge Thermal resistance), PWTR (Plane Wedge Thermal Resistance) and the conventional models. It is found that in the situation of shorter cylindrical tanks where the ratio between height and radius $H/R_2 < 10$, the errors generated by the RPSWT model are positive in most cases, with only a few exceptions, and the errors generated by the conventional model are negative in all cases. Thus, a new CRPSWTC model is proposed, which combines the RPSWT and conventional models with appropriate proportion factors to neutralize the positive and negative errors. The combination allows the new model to obtain very accurate results in comparison with the numerical solutions within this H/R_2 range. Nevertheless, the CRPSWTC model is proven to be applied to cases with larger H/R_2 and still obtain satisfactory results. Alternatively, the RPSWT model obtains the best results when $10 \leqq H/R_2 < 16$, while the PWTR model returns better solutions when $H/R_2 \geqq 16$.

Keywords: Insulation; Cylindrical tank; RPSWT model; PCTR model; PTR model; PWTR model

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1. Introduction

The heat transfer and thermal characteristics of cylindrical tanks have long been important research subjects due to their massive numbers of domestic and industrial applications. For example, Sun and Marrero [1] presented an experimental study on simultaneous heat and moisture transfer around a short porous cylinder during convection drying by a psychrometric method. Ghisalberti and Kondjoyan [2] studied the convective heat transfer coefficients between air flow and a short cylinder. Among the applications, the insulation problem of a hot or cold cylindrical fluid storage tank is especially important to our daily life and engineering applications since most hot or cold fluid storage tanks are in the shape of short cylinders. Conventionally, the PTR model [3,4] is applied to the top and bottom circular plates and the PCTR model [5,6] is used for the hollow cylindrical body to obtain the heat transfer characteristics of an insulated cylindrical tank. Recently, Chou and Wong [7] proposed a Plane Wedge Thermal Resistance (PWTR) model aimed to investigate the heat transfer characteristics associated with insulated polygonal pipes. In this model, the thermal resistances due to the inner convection term and the pipe conduction term were not considered, hence theoretically, its results were only reliable when applied to pipes with high inner convection coefficient h_i and high pipe conductivity K, such as condensers and evaporators. However, Wong et al. [8] proved that the PWTR model can be used in cases involving low to medium values of h_i and K, extensively enlarging the model's applicability to practical situations. Furthermore, Wong and Chou [9] also developed another Regular Polygon top Solid Wedge Thermal resistance (RPSWT) model to study the same heat transfer problem of insulated regular polyhedrons. The thermal resistance of the inner convection term and the wall conduction term were again neglected, and hence, again, the model should be only reli-

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Nomenclature

A_1	total inner surface area of cylindrical tank; $A_1 = A_{T1} + A_{C1}$
A_2	total outer surface area of bare cylindrical tank; $A_2 = A_{T2} + A_{C2}$
A_3	total outer surface area of insulated cylindrical tank; $A_3 = A_{T3} + A_{C3}$
A_{C1}	inner surface area of short cylindrical body
A_{C2}	outer surface area of bare cylindrical body
A_{C3}	outer surface area of insulated cylindrical body
A_{T1}	inner top and bottom surface area of cylindrical tank
A_{T2}	outer top and bottom surface area of bare cylindrical tank
A_{T3}	outer top and bottom surface area of insulated cylindrical tank
CRPSWTC	combined RPSWT model and conventional model
E_C	error generated by conventional model
E_M	error generated by CRPSWTC model
E_S	error generated by RPSWT model
E_W	error generated by PWTR model
H	height of cylindrical tank
h_i	inner heat convection coefficient
h_o	outer heat convection coefficient
J	dimensionless factor representing inner heat convection and conduction resistance effects; $J = (R_2/R_1)(h_o/h_i) + (R_2 h_o/K)\ln(R_2/R_1)$
K	thermal conductivity of wall
K_S	thermal conductivity of insulation layer
PCTR	plane cylindrical thermal resistance $[R_{th}]_C = \ln(R_3/R_2)/[2\pi K_s H]$
PWTR	plane wedge thermal resistance; $[R_{th}]_W = t^{*}\ln(A_3/A_2)/[K_s(A_3 - A_2)]$
PTR	plate thermal resistance; $[R_{th}]_P = t/(K_s A_2)$
Q_C	heat transfer rate of conventional model
Q_F	heat transfer rate from numerical computation
Q_M	heat transfer rate of CRPSWTC model
Q_S	heat transfer rate of RPSWT model
Q_W	heat transfer rate of PWTR model
RPSWT	regular polygon top wedge thermal resistance; $[R_{th}]_S = t/[K_s(A_3 A_2)^{1/2}]$
R_1	inner radius of cylindrical tank
R_2	outer radius of bare cylindrical tank
R_3	outer radius of insulated cylindrical tank
$(\sum R_{th})_B$	total thermal resistance of insulated cylindrical body calculated by PCTR model
$(\sum R_{th})_C$	total thermal resistance of insulated cylindrical tank with conventional model
$(\sum R_{th})_{P2}$	total thermal resistance of top and bottom circular plates of insulated cylindrical tank with PTR model
$(\sum R_{th})_S$	total thermal resistance of insulated cylindrical tank with RPSWT model
$(\sum R_{th})_W$	total thermal resistance of insulated cylindrical tank with PWTR model
s	distance along surface from centerline of insulated cylinder
t	thickness of insulation layer
t_1	thickness of wall
T_i	temperature of fluid inside cylindrical tank
T_o	temperature of fluid outside cylindrical tank
T_b	surface temperature on cylindrical body by PCTR model
$(T_{bn})_{max}$, $(T_{bn})_{min}$	maximum or minimum surface temperature on cylindrical body by numerical method
T_p	surface temperature on top or bottom plates by PTR model
$(T_{pn})_{max}$, $(T_{pn})_{min}$	maximum or minimum surface temperature on top or bottom plates by numerical method
x-, y-, z-	x-, y- and z-directions of Cartesian coordinate system

Greek symbol

α	proportion factors in CRPSWTC model; $\alpha = 1/[2.0 + H/R_2]$

able when applied to insulated regular polyhedrons with high h_i and K. Later, Lee et al. [10] demonstrated that the RPSWT model can indeed be successfully applied to insulated regular polyhedrons with low to medium values of h_i and K.

When the one dimensional PWTR model [7,8] is used to calculate a two dimensional heat transfer problem of an insulated regular polygonal pipe, poorer accuracy was returned with fewer polygon edges. Very recently, Wong et al. [11] found a reliable one dimensional analytical method to apply to the two dimensional heat transfer problem of an insulated rectangular duct. Their study showed that the PTR model under estimates the heat transfer rate, while the PWTR model over estimates this quantity at all test conditions. Thus, they developed the so-called Combined Plate Wedge Thermal Resistance (CPWTR) model that weighs the solutions of the PTR and PWTR models with proportion factors of 0.6 versus 0.4, respectively. This model has proven to yield highly accurate solutions for insulated rectangular duct problems. The present paper examines the possibility of employing similar practices in two dimensional insulated cylindrical tanks. Most domestic hot water tanks and many industrial fluid storage tanks are made in shapes of cylindrical tanks; hence the outcomes of the present investigation will be very useful to these applications.