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Quest for a mass-weighted-averaging turbulence theory, with an unexpected finding about the countergradient diffusion and blockage of heat and matters

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Abstract

A mass-weighted-averaging theory is formulated of density-variable inhomogeneous turbulence. There the velocity and the concentration of matters are resolved into the mass-weighted means and the fluctuations around them. The essence of the formulation lies in the introduction of mass-weighted fluctuations. With those new variables, a statistical theory of inhomogeneous incompressible turbulence can be extended to the variable-density counterpart in a straightforward manner. One of the unexpected findings by this theory is the striking similarity between the countergradient diffusion in a turbulent combustion and the heat-transport suppression in high-mode confinements of tokamak's thermonuclear fusion.

Keywords: Turbulence theory; Mass-weighted averaging; Countergradient diffusion; Transport barrier

1. Introduction

Gradient diffusion based on turbulent viscosity or diffusivity is one of the fundamental concepts in the study of turbulent transports. There, the momentum and scalars such as heat and matters are transported much more efficiently from their high- to low-magnitude regions, compared with a laminar-flow state. A well-known instance in which the gradient-diffusion concept completely breaks down is the countergradient diffusion of reactants and products in turbulent combustion (its reviews are given in

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Libby and Williams, 1994; Bray, 1995). Those matters are transported in the direction parallel to the mean concentration gradients in a sharp contrast to the gradient diffusion, and entirely different approaches need to be sought for the mathematical modeling of such a process. The representative is the modeling by Bray, Moss, and Libby who use a bimodal probability density function for the progressive variable (Libby and Bray, 1981).

The representative of a statistical theory of the countergradient diffusion is the work by Shimomura (1999) using the two-scale direct-interaction approximation (TSDIA) (Yoshizawa, 1984, 1998). There, the compressibility parameter expressing the degree of density variation is introduced, and various flow quantities are expanded in terms of the parameter. As a result, the TSDIA evaluation of the turbulent heat flux may be performed within the standard ensemble-averaging framework.

The mechanism of the countergradient diffusion has been investigated in detail by the direct numerical simulation (DNS) of a premixed planar flame freely propagating into isotropic turbulence with single-step chemistry (Rutland and Trouvé, 1993; Veynante et al., 1997; Veynante and Poinsot, 1997; Nishiki et al., 2002). These simulations revealed that the countergradient diffusion of reactants is linked with the thermal dilatation due to chemical reaction, whereas the usual gradient diffusion occurs when turbulent motion is dominant near a flame.

The countergradient diffusion is frequently encountered in a turbulent premixed flame. Such diffusion may be regarded as a situation in which the transport of matters is excessively suppressed and is reversed. The phenomenon of counteragent diffusion, however, seems to be quite curious to researchers in thermonuclear fusion. They have been long struggling for the blockage of the gradient diffusion of heat and particles.

Tokamaks confining plasmas in a torus are the most advanced method in thermonuclear fusion (Wessen, 1997). The discovery of high-confinement (H) modes has opened a new era in the study of plasma confinement (Wagner et al., 1982). In the modes, the transport of heat and particles is highly suppressed at the edge of plasmas. This phenomenon is called the formation of an edge transport barrier. H modes are characterized by the concurrence of radial electric field and poloidal plasma rotation in the close vicinity of the plasma edge (the poloidal rotation signifies the circumferential rotation in the cross section of a torus). The radial electric field is linked with the poloidal plasma rotation through the so-called $E \times B$ mechanism. The elucidation of the role of radial electric field has been one of the primary themes in the study of H modes since the pioneering works by Itoh and Itoh (1988) and Shaing and Crume (1989). The theoretical studies of H modes are reviewed by Itoh and Itoh (1996), Connor and Wilson (2000), and Yoshizawa et al. (2003).

At this stage, two questions occur: Why does the countergradient diffusion occur in turbulent combustion? and Is there any relationship between the mechanisms of the countergradient diffusion and the H-mode transport barrier? The proper answers to these questions are expected to be instrumental to the study of both turbulent combustion and thermonuclear fusion.

Variation of mass density is important in turbulent combustion and H modes. Then the construction of a turbulence theory for a variable-density flow is indispensable for clarifying the mechanisms in these phenomena. The representative method of statistically describing a variable-density turbulent flow is the use of the mass-weighted (Favre) averaging (Chassaing et al., 2002; Peters, 2000). This method is also familiar in the study of aeronautical turbulent flows. Nevertheless, the construction of a turbulence theory based on the averaging has long remained at a premature stage.

The author (Yoshizawa, 2003a) has quested for a mass-weighted turbulence theory and has very recently presented a theoretical method of overcoming the mathematical difficulty arising from the variation of