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Improved model for interfacial stresses accounting for the effect of shear deformation in plated beams

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ABSTRACT

A significant increase in strength and performance of reinforced concrete, timber and metal beams may be achieved by adhesively bonding a fibre reinforced polymer composite, or metallic such as steel plate to the tension face of a beam. One of the major failure modes in these plated beams is the debonding of the plate from the original beam in a brittle manner. This is commonly attributed to the interfacial stresses between the adherends whose quantification has led to the development of many analytical solutions over the last two decades. The adherends are subjected to axial, bending and shear deformations. However, most analytical solutions have neglected the effect of shear deformation in adherends. Few solutions consider this effect approximately but are limited to one or two specific loading conditions. This paper presents a more rigorous solution for interfacial stresses in plated beams under an arbitrary loading with the shear deformation of the adherends duly considered in closed form using Timoshenko's beam theory. The solution is general to linear elastic analysis of prismatic beams of arbitrary cross section under arbitrary loading with a plate of any thickness bonded either symmetrically or asymmetrically with respect to the span of the beam.

1. Introduction

Reinforced concrete (RC), metal or timber beams may be strengthened by adhesively bonding a fibre reinforced polymer (FRP) composite, steel or other metal plate to the soffit of the beam (Fig. 1). Such a strengthened beam is commonly termed a plated beam. This strengthening technique has become widely accepted in structural engineering for retrofitting and strengthening of existing structures, with FRP plating in particular being extensively employed. Under external loading, forces are transferred between beam and plate, generating interfacial shear and normal stresses in the adhesive layer between the adherends. Their concentration is highest at the plate ends due to the presence of a geometric discontinuity and their combination is believed to be responsible for the brittle debonding mode of failure commonly observed in tests which occurs well before the full flexural strength of the plated beam is reached.

Consequently, the interfacial stresses between the plate and the original beam have attracted a great interest in the last two decades and many analytical solutions [1–21] have been developed to quantify them. Among them, Smith and Teng [8] is simple, accurate and the most popular. All except Narayanamurthy et al. [20] and Zhang and Teng [21] are applicable only to one or a few specific loading conditions. Both are applicable to any loading arrangement and are simpler than and retain the accuracy of Smith and Teng [8]. All these but [7,9,13,14] are considered to be 'classical' solutions as they assume invariant stresses through the thickness of the adhesive layer and hence violate the free surface condition at the ends of the adhesive layer. However, as this affects the solution only within a few millimetres from the plate ends [2], classical solutions still offer useful insights of the behaviour of plated beams.

Higher order solutions [7,9,13,14] consider varying stresses through the thickness of the adhesive layer and satisfy the stress-free condition at the plate ends. However, they are complex to develop and difficult to adopt in practice because of stress singularity at the bi-material (adhesive and substrate) interface at the plate ends. This paper derives a closed-form solution similar to the classical solutions but which includes shear deformations in the adherends. A higher order solution is beyond the scope of this paper.

The adherends in a plated beam are generally subjected to axial, bending and shear deformations under external loading. However, most theoretical solutions neglect the effect of shear deformation of the adherends. Liu and Zhu [4] considered the effect of shear deformation of the beam only in their general solution of interfacial shear stress but provided an incomplete solution, omitting expressions for

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Notation

A	cross sectional area of the adhesive or adherends
b	width of the adhesive or adherends
E	modulus of elasticity of the adhesive or adherends
G	shear modulus of the adhesive
I	second moment of area of the adhesive or adherends about their centroidal axis
I_{1c}, I_{ac}, I_{2c}	second moment of area of beam, adhesive and plate section about the centroidal axis of the composite beam section respectively
I_e	second moment of area of the equivalent composite beam section about its centroidal axis
L	length of the adhesive or adherends
L_p	length of the plate
M	bending moment in the adherends
$M(0)$, $M(L_p)$	bending moment in plated beam at $x=0$ and $x=L_p$ under original loading ignoring the effects of plate end loading (Case-2)
$M_1(0)$, $M_1(L_p)$	bending moment in beam at $x=0$ and $x=L_p$ in Case-3 loading
$M_T(x)$	total applied bending moment at any section of the plated beam
N	axial force in the adherends
$N(x)$	resultant axial force resisted by any section of the adherends
$Q_e(x,y)$	first moment of area of equivalent adhesive or plate section about the centroidal axis of the composite beam section
t	thickness of the adhesive or adherends
u	longitudinal displacement of the adherends
v	vertical displacement of the adherends
$V(x)$	shear force at any section of adherends
$V_{Tc}(x)$	total applied shear force at any section of the composite beam
V_T	total shear force at any section of the plated beam in Case-3 loading
y_c	vertical distance from top of the beam to the centroid of the composite beam section
y_1, y_2	vertical distance from bottom of the beam and top of the plate to their respective centroids respectively
pl, pr	subscripts referring respectively to the left and right end of the plate
1, a, 2	subscripts referring respectively to the beam, adhesive and plate
κ_i	Timoshenko's shear coefficient
$\sigma(x)$	interfacial normal stress at any section of the plated beam
$\tau(x)$	interfacial shear stress at any section of the plated beam
γ_{xy}	engineering shear strain at the adhesive layer
$\varepsilon_1(x)$, $\varepsilon_2(x)$	longitudinal strain at bottom layer of beam and at top layer of plate respectively
ψ_1–ψ_6	roots for the governing differential equation of $\tau(x)$ in Case-3 loading
η_1–η_5	roots for the governing differential equation of $\sigma(x)$ in Case-3 loading
B_1–B_{12}	constants of integration in general solution of $\tau(x)$ in Case-3 loading and
C_1–C_6	constants of integration in general solution of $\sigma(x)$ in Case-3 loading

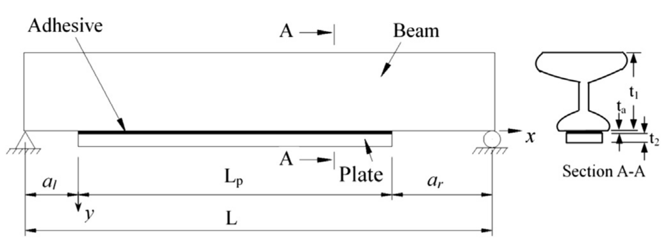

Fig. 1. Plated beam.

the constants of integration. Smith and Teng [8] considered the shear deformation of the adherends within the governing differential equations but neglected it when deriving the general solutions to avoid complexities in obtaining general solutions from the two strongly coupled governing equations. Abdelouahed [17,22] (applicable to UDL and single point loads) and Yang and Wu [18] (applicable to UDL only) adopted the solution of Smith and Teng [8] and included the shear deformation effect only approximately in the solution of interfacial shear stress. Their solutions suggest that the effect of shear deformation predominates on interfacial shear stress but is negligible for interfacial normal stress, although the present solution and finite element (FE) predictions will demonstrate that this is not necessarily correct. Narayanamurthy et al. [23] (applicable to all loading arrangements) include an approximation to the effect of shear deformation on the interfacial shear stress and used Timoshenko's beam theory to derive interfacial normal stress. This is the first closed-form solution that included the effects of adherend's shear deformation on both interfacial shear and normal stresses in plated beams. Although the formulation for interfacial normal stress is accurate, its accuracy is compromised by the approximation involved in interfacial shear stress. These four solutions have adopted different approximations to overcome mathematical difficulties in arriving at their general solutions. Recently Edalati and Irani [24] provided a solution applicable only to a UDL by considering all three deformations in adherends in closed-form but it predicted a reduction in interfacial normal stress when compared with that of Smith and Teng [8], the opposite trend to that predicted by FE analyses as well as by the solution presented here. The actual effect of adherends' shear deformation is not yet clearly understood.

The effect of shear deformations has been investigated in adhesively bonded single and double lap joints subjected to axial loading. Delale et al. [25] modelled the stresses in single lap joints made of orthotropic adherends which were assumed to be very thin compared to the lateral dimensions and used Reissner's plate theory in a plane strain state. Tsai et al. [26] considered the shear deformation in double lap joints under tension by assuming a linear shear stress variation through the thickness of the adherends and treated the adherends as two thin beams, instead of thin plates as in Delale et al. [25]. Many other important interfacial stress solutions for double lap joints under axial loading are reviewed in detail by Chalkley and Rose [27]. These solutions highlight that the accuracy of the predicted interfacial stresses can be improved by considering the shear deformations of the adherends, but they clearly cannot be directly applied to the present problem because of the differences in both structural form and loading conditions.

Further, the shear deformation effect in multilayered composites and sandwich plates loaded by transverse pressure with various in-plane distributions has been studied by Carerra and Ciuffreda [28] using a unified formulation. They compared about 40 theories based on equivalent single layer models and layer-wise models within the framework of principle of virtual displacement and Reissner's mixed variational theorem and presented a closed form solution for orthotropic plates by expanding the applied pressure loading in Fourier