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An elastic solution for adhesive stresses in multi-material cylindrical joints

S. Kumar [a,b,*], M.A. Khan [a]

[a] Institute Center for Energy (iEnergy), Department of Mechanical and Materials Engineering, Masdar Institute of Science and Technology, PO Box 54224, Abu Dhabi, UAE
[b] Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139-4307, United States

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ABSTRACT

The stress analysis of cylindrical adhesive joints is less frequently reported in the literature compared with that of flat geometries. Determination of shear and peel stresses in the adhesive layer is critical to the design of such multi-material joints. In this study, a theoretical framework is presented for the stress analysis of shaft-tube adhesive joints subjected to axial tensile loads. The axisymmetric assembly consists of similar or dissimilar isotropic or orthotropic adherends and a homogeneous adhesive layer. Adopting a generic stress function approach, stress fields are derived by enforcing the traction and traction-free boundary conditions as well as the stress continuity conditions at the interfaces. The principle of minimum complementary energy in conjunction with a variational method is used to obtain the governing differential equations in order to determine the stress-state in each of the constituents. Shear and peel stress distribution within the adhesive layer is presented to showcase the presence of stress concentration at the ends of the overlap. To verify the accuracy of the theoretical results, an identical axisymmetric finite element model is created and Finite Element (FE) analysis is performed. The stress fields obtained from the analytical model are in good agreement with the FE predictions. The influence of overlap length and stiffness mismatch between the adherends on peak adhesive stresses and their distribution are studied through a systematic parametric study. The findings of this study provides insight into the optimal design of multi-material adhesive joints.

1. Introduction

Adhesive joints are lighter and they provide more uniform load transfer between the adherends by reducing the stress concentration. The design of adhesive joints is greatly influenced by the peak shear and peel stresses in the adhesive interlayer. Numerical and analytical studies have been conducted to determine the stress-state in the adhesively bonded assemblies [1–24]. Owing to the complexity of the exact physical problem, efforts have been made to analyse adhesive joints incorporating simplistic assumptions. Initial insight on the behaviour of adhesive-lap joint was first provided by Goland and Reissner (G–R) [1,2]. They studied the effect of adhesive peel stress and adherend bending due to load-path eccentricity. Subsequently, Lubkin and Reissner (L–R) [2] developed an analytical solution for axially loaded tubular joints considering tension, shear and bending of adherends. However, L–R model only considered shear and peel stresses as a function of axial coordinate ignoring axial and hoop stresses in the adhesive layer. Hart–Smith's [3] attempt to reduce discrepancies in G–R's solution led to closed form solution which considered adhesive's plasticity and adherends stiffness imbalance. Hart–Smith (H–S) reported that adhesive plasticity eliminates shear failure mode in most cases.

Axisymmetric FE analysis of tubular lap joints subjected to axial and torsional loads were performed by Adam and Peppiatt [4] to predict the variation of stress distribution due to the presence of an adhesive fillet. Delale et al. [7] employed plate theory to analyse adhesively bonded joints but considering transverse shear in the adherends and in-plane normal strains in the adhesive. Similar to the FE analysis of Adam and Peppiatt [4], Nagaraja and Alwar [6] performed FE analysis of tubular adhesive-lap joints considering non-linear stress–strain behaviour of the adhesive and demonstrated that the linear elastic constitutive model for the adhesive overestimates the stresses. They subsequently studied the

* Corresponding author at: Institute Center for Energy (iEnergy), Department of Mechanical and Materials Engineering, Masdar Institute of Science and Technology, PO Box 54224, Abu Dhabi, UAE. Tel.: +971 2 810 9239; fax: +971 2 698 8121.
E-mail addresses: kshanmugam@masdar.ac.ae, s.kumar@eng.oxon.org (S. Kumar).

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Nomenclature

P	Applied axial tensile load
σ_0	Stress applied corresponding to 'P' on the shaft adherend
b, c, d	Radius of the shaft, outer radius of the adhesive and outer radius of the tube, respectively
$\rho=(d^2-c^2)/b^2$	Area ratio of the tube and the shaft
L	Overlap length of the joint
$\eta=z/L$	Non-dimensional distance over the bond length from the origin
(r,θ,z)	Cylindrical coordinates system representing radial, circumferential and longitudinal directions respectively
$E_{ti}, E_{li}, \nu_{ti}, \nu_{tli}, \nu_{lti}, G_{tli}$	Young's modulus, Poisson's ratio, and shear modulus of transversely isotropic member 'i'. l and t denote longitudinial and transverse directions, respectively.
$\phi_i(r,z)$	Stress function for the member 'i'.
$\sigma_{zz}^{(i)}(r,z)$	Stress in the longitudinal direction in member 'i'
$\sigma_{rr}^{(i)}(r,z)$	Radial stress component in member 'i'
$\sigma_{\theta\theta}^{(i)}(r,z)$	Circumferential stress component in member 'i'
$\sigma_{rz}^{(i)}(r,z)$	Shear stress component in member 'i'
$\sigma_{mn}^{(i)}(r,z)$	Stress tensor at a material point of member 'i'
$\epsilon_{mn}^{(i)}(r,z)$	Strain tensor at a material point of member 'i'
$\alpha_{mn}^{(i)}(r,z)$	Coefficient of linear thermal expansion tensor of member 'i'
ΔT	Uniform change of temperature from a stress-free state
$\Pi^{(i)}(r,z)$	Strain energy density in member 'i'
$U^{(i)}(r,z)$	Strain energy in member 'i'
Ψ	Energy functional of the bonded system
$f_i(r), g_i(z), h_i(z)$	Unknown functions used to define stress function ϕ_i
f_i'	Derivative of f_i with respect to 'r'
g_i'	Derivative of g_i with respect to 'z'
h_i'	Derivative of h_i with respect to 'z'
K_q	Unknown functions or constants that satisfy boundary conditions ($q=1\ldots6$)
A_i, B_j, C_k, D_s	Constants that depend on material and geometric properties as well as loading conditions ($i=1\ldots7, j=1\ldots18, k=1\ldots4, s=1\ldots18$)

Subscript/Superscript

i	Variable number 1 for shaft, 2 for adhesive, 3 for tube
m, n	Independently range over r,θ,z

viscoelastic response of adhesively bonded lap-joints by FE method and reported that the adhesive stresses significantly decrease due to viscous effect [5]. Furthermore, linear viscoelastic analysis of adhesive-lap joints performed by Delale and Erdogan [8] indicates that the peak shear stresses are not only less than those of normal stresses but also decays at a faster rate.

Pickett and Hollaway [9] analysed single, double and tubular lap-joint configurations using two different adhesive constitutive models, viz., 1. elastic-perfectly plastic and 2. nonlinear elasto-plastic. Strength prediction based on plastic shear strain failure criterion was adopted as it was dominant in double and tubular joint configurations. Fully elastic analysis of lap joints with two parameter formula was given by Bigwood and Crocombe [10]. Oplinger [11] studied the effect of adherends' deflection of single lap-joint using beam theory considering bond shear strains and compared the results with those of classical G–R solution and H–S models. Oplinger predicted that for thicker adherends, G–R solution yields accurate results unlike H–S solution as it neglects bending effects.

Elasto-viscoplastic behavior of the adhesive with pressure sensitive von Mises yield criterion along with geometrically nonlinear response of the joint was considered by Pandey and his coworkers. Their study indicates that combined material and geometric nonlinear behavior reduces the peak peel and shear stresses [12]. da Silva et al. [25] reviewed the analytical models of single-lap joints and noted that most of the analytical studies reported in the literature thus far considered 2D plane strain formulation with linear elastic constitutive behaviour. This assumption of linear elastic behavior of the adhesive is justified by the fact that viscoelastic behavior does not affect the stress distribution but reduces peak shear and peel stresses as reported by Pandey and his co-workers. Kim et al. [14] performed experiments on tubular adhesive-lap joints and compared the results with those of non-linear FE analysis. They found that the strength variation with thickness of the adhesive depends on the residual stresses. Pugno and Carpinteri [15] analysed tubular adhesive joints for static and dynamic load cases focussing mainly on possible theoretical joint profiles which give uniform axial and/or torsional strengths. Their study indicates that uniform strength reduces the weight and increases the strength of joints. Recently, an explicit closed form solution for L–R model to predict the adhesive stresses in axially loaded cylindrical joints was provided by Goglio and Paolino [26].

Shi and Cheng [13] were the first ones to provide theoretical insight into the analysis of cylindrical assemblies. By neglecting the axial stress in the adhesive interlayer, stress fields were determined by exactly satisfying traction-free boundary conditions and stress continuity conditions. Nemes and Lachaud [16] used a similar approach and developed a simplified model yet again neglecting the axial stress in the adhesive layer. Kumar et al. investigated the behaviour of axisymmetric adhesive joints with material tailored interlayer [17–21,27,28,22,23]. Their solution assumes constant radial stress in the adhesive interlayer. Aforementioned models account for traction-free boundary conditions but neglect axial stress in the adhesive interlayer. However, recent studies indicate that the axial stress in the adhesive layer significantly influences the failure behaviour of the bonded system [17,29]. The purpose of this study is to determine the complete stress field in the entire assembly with a particular focus on shear and peel adhesive stresses including the axial stress in the adhesive layer, while exactly satisfying traction and traction-free boundary conditions. Therefore, in this study, a stress function approach is adopted in conjunction with the principle of minimum complementary energy (PMCE) to develop a tractable analytical model for cylindrical joints experiencing axial tension. However, it should be noted that this study is not focused on singular stress fields that arise at the free surface of interfaces between the adherends and the adhesive.

2. Analytical model

Consider a cylindrical joint consisting of an isotropic or a transversely isotropic shaft adhered to a transversely isotropic or an isotropic tube by an adhesive interlayer. The bonded assembly is subjected to an axial tensile load P as shown in Fig. 1a. σ_0 is the stress applied to the shaft corresponding to 'P'. This system is referred to a cylindrical coordinate system (r,θ,z) as shown in