International Journal of Adhesion & Adhesives 39 (2012) 54–59

Contents lists available at SciVerse ScienceDirect

International Journal of Adhesion & Adhesives

journal homepage: www.elsevier.com/locate/ijadhadh

A straightforward method to obtain the cohesive laws of bonded joints under mode I loading

M.F.S.F. de Moura [a],*, J.P.M. Gonçalves [b], A.G. Magalhães [c]

[a] Faculdade de Engenharia da Universidade do Porto, R. Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
[b] IBM T. J. Watson Research Center, 1101 Kitchawan Road, Yorktown Heights, NY 10598, USA
[c] Instituto Superior de Engenharia do Porto, R. Dr. António Bernardino de Almeida, 431, 4200-072 Porto, Portugal

ARTICLE INFO

Article history:
Accepted 14 July 2012
Available online 20 July 2012

Keywords:
Bonded joints
Cohesive laws
Mode I
Double cantilever beam test

ABSTRACT

A simple procedure to measure the cohesive laws of bonded joints under mode I loading using the double cantilever beam test is proposed. The method only requires recording the applied load–displacement data and measuring the crack opening displacement at its tip in the course of the experimental test. The strain energy release rate is obtained by a procedure involving the Timoshenko beam theory, the specimen's compliance and the crack equivalent concept. Following the proposed approach the influence of the fracture process zone is taken into account which is fundamental for an accurate estimation of the failure process details. The cohesive law is obtained by differentiation of the strain energy release rate as a function of the crack opening displacement. The model was validated numerically considering three representative cohesive laws. Numerical simulations using finite element analysis including cohesive zone modeling were performed. The good agreement between the inputted and resulting laws for all the cases considered validates the model. An experimental confirmation was also performed by comparing the numerical and experimental load–displacement curves. The numerical load–displacement curves were obtained by adjusting typical cohesive laws to the ones measured experimentally following the proposed approach and using finite element analysis including cohesive zone modeling. Once again, good agreement was obtained in the comparisons thus demonstrating the good performance of the proposed methodology.

1. Introduction

The use of bonded joints in structural applications has been increasing in the most recent years as a consequence of the advantages of this joining method relative to classical alternatives, as is the case of fastening. The main advantages are lower weight, less sources of stress concentration and better fatigue properties. The application of this joining method in transportation industries, like automobile and aeronautical, requires more demanding design methods in order to better describe the mechanical behavior of the bonded joints. In fact, classical approaches based on stress or strain analysis are not able to deal with several details influencing the mechanical behavior of the bonded joints. For example, when these methods are applied through finite element analysis it is verified that mesh dependency problems arise, due to the presence of singularities. In this context, cohesive zone models (CZM) emerge as an appealing alternative solution. These methods are based on a constitutive relationship between stresses (σ) and relative displacements (w) and allow simulation of damage initiation and propagation. They use the strength of materials approach to identify damage onset and fracture mechanics concepts to deal with its growth. The CZM are usually implemented in a finite element analysis by means of interface finite elements connecting solid elements [1]. Some issues, like non-self-similar crack growth and the presence of a non-negligible fracture process zone (FPZ), are well managed by CZM. The FPZ is the region in the vicinity of the crack tip where plasticity, micro-cracking and several other inelastic processes take place. When ductile adhesives are used, the size of the FPZ is non-negligible and its incorporation in the predictive method is fundamental to provide reliable design [2,3].

One of the crucial aspects of CZM is the definition of the cohesive law that characterizes the bonded joint. There are two main methods to get these laws: inverse method and direct measurement during a fracture characterization test. The inverse method assumes a pre-defined cohesive law and the respective parameters are determined by fitting the numerical and experimental load–displacement curves using a manual iterative procedure [4] or an automatic optimization strategy [5]. The drawback intrinsic to this procedure is the need to impose a pre-defined law. In fact, this task

---

* Corresponding author. Tel.: +351 225081591; fax: +351 225081727.
E-mail address: mfmoura@fe.up.pt (M.F.S.F. de Moura).

0143-7496/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijadhadh.2012.07.008

requires some previous knowledge of the joint's behavior which is not available in many cases.

Alternatively, the cohesive law can be measured directly during a fracture characterization test. Sørensen [6] determined experimentally the cohesive law by means of a J-integral based approach. The author performed double cantilever beam (DCB) tests where the specimens were loaded with pure bending moments which required the development of a special experimental setup. Using this procedure, the J-integral can be calculated continuously during the test as a function of the applied moment using a simple closed-form solution. The end–opening displacement (w) at the crack tip was monitored by extensometers mounted at pins located at the neutral axis of the specimen arms. The cohesive law ($\sigma = \mathrm{f}(w)$) is obtained from differentiation of J with respect to w. Andersson and Stigh [7] obtained the stress–elongation relation for an adhesive layer loaded in peel using the DCB test. These authors used a common test configuration to perform the DCB tests, i.e., the specimens were loaded with a wedge force. However, in this case the J-integral estimation required the measurement of the beam rotation at the loading point by means of a specific shaft encoder thus allowing the determination of J-integral by using a closed-form solution.

The objective of this work is to present a simpler methodology to determine the cohesive laws of bonded joints under mode I loading using the DCB test. The method only involves the data given by the load–displacement curve and monitoring of the crack opening displacement (COD) at the crack tip. Evolution of the specimen's compliance during the experimental test is used in combination with the Timoshenko beam theory and the equivalent crack concept to determine the strain energy release rate. The cohesive law is obtained by the derivative of the strain energy release rate as a function of the COD. Following this procedure it is not necessary to employ neither a specific experimental setup nor the measurement of the specimen's arms rotation during the course of the test. The proposed method is validated numerically by means of a finite element analysis including cohesive zone modeling and also experimentally, performing DCB tests on steel-epoxy bonded joints.

2. Model description

The proposed model is applied to evaluate the cohesive law of a bonded joint under mode I loading using the DCB test. The method is based on direct measurement of the COD (represented in equations by w) at the crack tip and on the evaluation of the J-integral or energy release rate by a procedure which is different from the approaches presented in the literature. The cohesive law ($\sigma = \mathrm{f}(w)$) can be obtained from differentiation of the following Eq. [8]:

$$J_\mathrm{I} = \int_0^w \sigma(w)\, dw \tag{1}$$

leading to

$$\sigma(w) = \frac{dJ_\mathrm{I}}{dw} \tag{2}$$

This means that obtaining the J_I–w relation is a crucial issue of the procedure. In this context, a method based on the specimen's compliance, the Timoshenko beam theory and the crack equivalent concept is presented to estimate the evolution of strain energy release rate as a function of w.

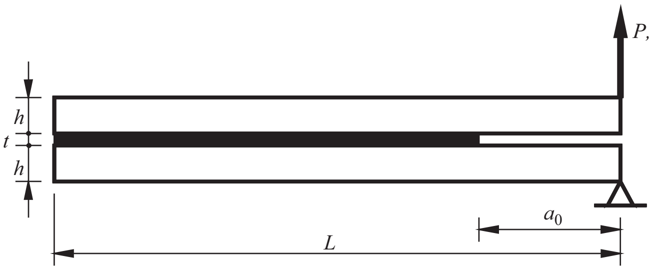

Fig. 1. Schematic representation of the DCB test ($L=120$, $a_0=40$, $h=3$, and $t=0.2$; specimen width $B=25$; all dimensions in mm).

According to the Timoshenko beam theory, the compliance versus crack length relationship ($C=\mathrm{f}(a)$) considering isotropic adherends is [5]

$$C = \frac{8a}{EBh}\left(\frac{a^2}{h^2} + \frac{3(1+\nu)}{5}\right) \tag{3}$$

where a is the crack length, B and h are the specimen's width and the adherend's height, respectively (Fig. 1), and ν is Poisson's ratio. An equivalent elastic modulus (E_e), accounting for the combined effects of adherends and adhesive, specimen variability and stress concentrations at the crack tip, can be obtained from the previous equation taking into account the initial conditions. Considering the initial compliance C_0 obtained from the early linear part of the load-displacement curve and initial crack length a_0 corrected to account for root rotation effects, the E_e becomes

$$E_\mathrm{e} = \frac{8(a_0+\Delta)}{C_0 Bh}\left(\frac{(a_0+\Delta)^2}{h^2} + \frac{3(1+\nu)}{5}\right) \tag{4}$$

The crack length correction Δ can be obtained numerically for each specimen fitting the initial compliance C_0 with the experiments for the real a_0. Afterwards, two additional numerical analyses considering different initial crack lengths should be performed, thus defining three points in the graphic representation of the $C^{1/3}=\mathrm{f}(a)$ relation. The interception of this line with the abscissa axis allows the definition of the crack length correction [4]. This modus operandi can also be executed experimentally considering three different initial crack lengths and performing the fracture characterization test from the smaller initial crack length. See details in Ref. [9].

During propagation, Eq. (3) can be used to estimate the equivalent crack length a_e as a function of the current compliance C. The resulting equation is

$$\frac{8a_\mathrm{e}^3}{E_\mathrm{e}Bh^3} + \frac{24(1+\nu)a_\mathrm{e}}{5E_\mathrm{e}Bh} - C = 0 \tag{5}$$

whose analytical solution [5] can be obtained from the Matlab® software. Using this procedure the FPZ effect is accounted for, since its presence influences the load–displacement curve, i.e., the compliance C, which is used to estimate the a_e. The strain energy release rate is obtained combining the Irwin-Kies equation

$$J_\mathrm{I} = \frac{P^2}{2B}\frac{dC}{da} \tag{6}$$

with Eq. (3) and considering the equivalent quantities a_e and E_e instead of a and E, respectively,

$$J_\mathrm{I} = \frac{12P^2}{E_\mathrm{e}B^2h}\left(\frac{a_\mathrm{e}^2}{h^2} + \frac{1+\nu}{5}\right) \tag{7}$$

Following this methodology the evolution of the strain energy release rate during the test is obtained exclusively by means of the data provided by the load–displacement curve, and in a straightforward manner combined with the measured COD. It is not necessary to monitor the crack length, since in this case the equivalent crack length is a calculated parameter as a function of the current compliance. It should be noted that a_e accounts