European Journal of Mechanics A/Solids 57 (2016) 165–177

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Discrete approaches for the nonlinear analysis of in plane loaded masonry walls: Molecular dynamic and static algorithm solutions

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ARTICLE INFO

Article history:
Received 26 February 2015
Accepted 21 December 2015
Available online 31 December 2015

Keywords:
Masonry like material
Discrete model
Nonlinear analysis

ABSTRACT

The aim of the paper is to present and validate a non commercial discrete element model (DEM) code for the nonlinear analysis of in plane loaded masonry panels, with dry or mortar joints. Such model is based on the hypothesis of rigid blocks and joints modeled as interfaces, that turn out to be both suitable for representing the behavior of ancient masonry, characterized by joint size negligible with respect to block size and block stiffness larger than joint stiffness. Hence, the elastic and inelastic behavior of a masonry assemblage is concentrated at joints by defining their stiffness and adopting a Mohr-Coulomb law as a restraint for interfacial actions. The proposed strategy is based on two approaches: a static solution method and a molecular dynamics algorithm. The static solution method allows to determine the stiffness matrix of a masonry panel and to update such matrix accounting for actual joint stiffness and blocks arrangement. Such method turns out to be computationally faster and equally effective with respect to the molecular dynamics one for performing incremental analysis of in plane loaded masonry panels. On the other hand, the molecular dynamics method is computationally less onerous than the static solution method, since it does not require to define and update panel stiffness matrix and to invert it for determining displacements. Both approaches are used and critically compared for solving several case studies of masonry panels modeled by DEM. In addition, it must be pointed out that results in terms of ultimate loads and collapse mechanisms are in good agreement with existing experimental data and numerical solutions.

1. Introduction

Discrete element models represent a class of numerical models that study the mechanical behavior of systems made of particles, blocks or multiple bodies, as stated in the work of Lemos (2007) dedicated to DEM applied to masonry structures. This model type is particularly adopted for modeling rocks, soil, concrete or masonry. Additional names that may be used in substitution of DEM are distinct elements, discontinuous deformation analysis (DDA), rigid body spring model (RBSM), discrete-finite elements. With a DEM is possible to study the behavior of distinct bodies, eventually subdivided into finite elements, subject to displacements and rotations and interacting each other by means of contact elements. For this purpose, the model is frequently formulated in the dynamic field and molecular dynamics algorithms are adopted for obtaining numerical solutions, starting by assigning a perturbation to the initial model and solving the equation of motion with a direct integration in the time domain.

Considering the field of masonry structures, it is well known that masonry is a heterogeneous structural material obtained by composition of blocks connected by dry or mortar joints. Particularity of this heterogeneous material is that the size of heterogeneity (size of block) may be not negligible with respect to the global size of the structural element as in several composite materials. For this reason, in the last decades, several researchers developed models for studying masonry-like material adopting different approaches.

A discrete element model, based on the assumptions of rigid block behavior and joints modeled as interfaces, may be suitable for investigating masonry behavior due to the small number of degrees of freedom (DOFs) needed for performing a numerical analysis of block assemblages. These assumptions may be suitable for historical masonry, in which block stiffness is larger than joint stiffness, allowing to assume blocks as rigid bodies; moreover joint thickness is negligible if compared with block size, especially in case of dry joints, allowing to model joints as interfaces. In the following, the

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http://dx.doi.org/10.1016/j.euromechsol.2015.12.008
0997-7538/

word 'interface' will be used for indicating the contact surface between adjacent blocks. With this model, elastic and inelastic deformations can only develop at joints; hence masonry is seen as a 'skeleton' in which blocks are connected by springs and the interactions between blocks are represented by forces and moments that depend on their relative displacements and rotations. As well known, historical masonry structures may be characterized by random textures and random size of blocks; in this work, for simplicity, attention is given to regular assemblages characterized by blocks having the same dimensions and equally spaced horizontal and vertical joints (namely the 'opus quadratum' described by Vitruvius). Moreover, one leaf masonry panels are considered, with only one block along panel thickness.

Discrete models were adopted in the past by many authors for studying masonry behavior in linear and nonlinear fields (Baggio and Trovalusci, 1993; Casolo, 2004, 2006, 2009; Ferris and Tin-Loi, 2001; Formica et al., 2002; Livesley, 1978; Masiani et al., 1995). In particular, Cecchi and Sab (2004, 2009) defined a simple and effective DEM for studying the three-dimensional behavior of masonry panels and for modeling regular and random brickwork. Recently, such model has been extended to the viscoelastic field (Baraldi and Cecchi, 2014a) and it has been reviewed and compared with continuum models for masonry structures (Baraldi et al., 2015a). An exhaustive description of discrete models and their improvement in several scientific fields up to recent years may be found in the work of Lemos (2007). Limits in DEM are mainly represented by rigid block assumptions, that were overcame taking into account the deformability of elements by introducing additional parameters (Itasca, 1989) or FE discretizations (Mahabadi et al., 2012; Munjiza, 2004). Moreover, DEM commercial code UDEC (Itasca, 1989) and FEM/DEM Y-GUI open source code (Munjiza, 2004) are all based on molecular dynamics algorithm that allows to perform static, dynamic, linear and nonlinear analysis but not modal analysis. Recently, a comparison between such models and a simple DEM has been carried on for studying masonry linear behavior (Baraldi et al., 2013) and nonlinear behavior (Baraldi et al., 2015b). The FEM/DEM code cited above was also adopted for modeling dry masonry structures in-plane loaded (Smoljanovic et al., 2013).

Due to the rigid block hypothesis, the nonlinear behavior of the model is concentrated at joints. In this work, an elastoplastic joint behavior is considered, by adopting a Mohr-Coulomb frictional law for restraining interfacial forces, similarly to the work of Ferris and Tin-Loi (2001), Trovalusci and Masiani (2003). Other types of nonlinear behavior may take into account elasticity, friction and a damage evolution law for the joints (Casolo, 2009; Formica et al., 2002; Gambarotta and Lagomarsino, 1997; Lofti, and Shing, 1994). In this field of analysis, Lourenço and several co-workers (Lourenço and Rots, 1997; Orduña and Lourenço, 2003, 2005; Milani and Lourenço, 2012) developed and improved a nonlinear model for masonry structures based on a modified Mohr-Coulomb yield criterion, accounting for five different local failure mechanisms and performing both pushover and limit analysis in plane state and three dimensions case.

The main objective of this work is to extend to the nonlinear frame the linear elastic DEM introduced by Cecchi and Sab (2004). For this purpose, a Mohr-Coulomb law is adopted for describing interface behavior, in order to perform incremental analysis of masonry panels and to determine their collapse load and mechanism. The second objective of the work is to present a fast and effective static solution method for performing in plane nonlinear incremental analysis with DEM, instead of adopting the molecular dynamics algorithm, which was the solution method originally proposed by authors (Cecchi and Sab, 2004). Such static solution method has recently turned out to be effective in the elastic field for the in plane modal analysis of masonry panels (Baraldi and Cecchi, 2014b). The proposed model and solution method is validated by solving several numerical examples and comparing results with existing experimental and/or numerical solutions (Baggio and Trovalusci, 1993; Ceradini, 1992; Formica et al., 2002; Page, 1978).

2. Discrete model

2.1. Introduction to discrete element modeling

Discrete element modeling represents an effective tool for assessing the mechanical behavior of materials or structures made of separate components such as rocks, stone blocks, bricks and grains. Any type of DEM is characterized by two components: elements and contacts. Elements can have different shapes and can be perfectly rigid, deformable by means of subdivisions into simple FEs or deformable by means of simple functions and strain parameters. Differently than FE analysis, elements in DEM can move independently, element displacements can be large and element contacts can vary during the analysis; for this reason contact detection algorithms are often adopted for identifying elements in contact. Contacts between elements are characterized by forces transmitted between them and several constitutive relations describe how to determine such forces. The determination of the displacements of the system represented by a DEM involves the solution of a system of equations of motion that is generally solved by means of dynamic (time integration) methods and considering each displacement separately. The DEM developed in geomechanics by Cundall and co-workers (Cundall, 1971; Cundall and Hart, 1992; Itasca, 1989) is characterized by all the features listed above (element discretization, contact forces depending on element overlapping and an explicit time step integration for determining displacements). Another example of discrete modeling, for the two-dimensional case, is the DDA (discontinuous deformation analysis, Shi, 1988), characterized by deformable elements described by the usual small strain tensor. The DEM adopted in this work is based on the original research of Cecchi and Sab (2004) and it is characterized by perfectly rigid elements having rectangular shape and forming a regular pattern, then the model involves few Lagrangian parameters and a relatively small system of equations of motion needs to be solved. Only face-to-face contacts are considered, depending only on relative displacements between adjacent elements and contact topology do not vary during analysis; hence, the adopted model does not require contact detection algorithm, moreover, contacts due to element overlapping are not considered. For these reasons the adopted model is simpler than other DEM types.

2.2. DEM with perfectly rigid elements and regular texture

A standard running bond periodic masonry is considered and Fig. 1a shows a Representative Elementary Volume (REV) of the masonry pattern, having a block $B_{i,j}$ surrounded by six blocks. Block plane dimensions are: a (height) and b (width), whereas s represents block and panel thickness. Analysis is performed in a linearized two-dimensional (2D) framework, and assuming rigid block hypothesis and 2D plane stress hypothesis, the displacement of each block $B_{i,j}$ is a rigid body motion referred to the motion of its center $\mathbf{y}^{i,j}$ and defined by the following expression:

$$\mathbf{u}^{i,j}(\mathbf{y}) = \mathbf{u}^{i,j} + \boldsymbol{\Omega}^{i,j}\left(\mathbf{y} - \mathbf{y}^{i,j}\right), \tag{1}$$

where $\mathbf{y}$ is a generic position on plane y_1y_2 for a point of block $B_{i,j}$, $\mathbf{u}^{i,j} = \{u_1^{i,j}\ u_2^{i,j}\}^T$ is the translation vector of the centre $\mathbf{y}^{i,j}$ of block $B_{i,j}$