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Computational method for excluding fibers under compression in modeling soft fibrous solids

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ABSTRACT

Soft fibrous solids often consist of a matrix reinforced by fibers that render the material anisotropic. Recently a fiber dispersion model was proposed on the basis of a weighted strain-energy function using an angular integration approach for both planar and three-dimensional fiber dispersions (G.A. Holzapfel and R.W. Ogden: Eur. J. Mech. A/Solids, 49 (2015) 561–569). This model allows the exclusion of fibers under compression. In the present study computational aspects of the model are documented. In particular, we provide expressions for the elasticity tensor and the integration boundary that admits only fibers which are extended. In addition, we give a brief description of the finite element implementation for both 2D and 3D models which make use of the von Mises distribution to describe the dispersion of the fibers. The performance and the finite element implementations of the 2D and 3D fiber dispersion models are illustrated by means of uniaxial extension in the mean fiber direction and more general directions, and simple shear with different mean fiber directions. The finite element results are in perfect agreement with the solutions computed from analytical formulas.

1. Introduction

In many soft fibrous solids, including biological tissues, there exists a matrix reinforced by embedded fibers which, in general, induce anisotropy in the material. For some materials the matrix can be treated as homogeneous and isotropic. The fibers may be distributed within the matrix in various ways. Specifically, in human arterial walls the collagen fibers are not perfectly aligned but are dispersed around a mean direction. Such a fiber dispersion has been observed in, for example, human arterial walls (Canham et al., 1989; Finlay et al., 1995, 1998; Schriefl et al., 2012; Schriefl et al., 2013), the myocardium (Karlon et al., 1998; Covell, 2008), corneas (Boote et al., 2004, 2005) and articular cartilage (Lilledahl et al., 2011). In particular, recent extensive experimental results (Schriefl et al., 2012) have shown that the collagen fiber dispersion in each of the layers of (healthy) human thoracic and abdominal aortas and iliac arteries is non-symmetric, in contrast to the rotationally symmetric fiber dispersion assumed in previous studies; see, for example, Gasser et al. (2006). In order to improve understanding of the mechanical properties of such tissues, constitutive modeling is essential.

Motivated by the specific structural arrangements of collagen fibers, various constitutive relations have been developed. Fiber dispersion has been represented in such constitutive relations either by direct incorporation in a strain-energy function via a probability density function (PDF) or by a generalized structure tensor. Following Cortes et al. (2010) these two approaches are referred to as ‘angular integration’ (AI) and ‘generalized structure tensor’ (GST), respectively. For a short survey of the main existing constitutive models that account for dispersion of collagen fibers by using either the AI approach (due to Lanir, 1993) or the GST approach, see the review in Holzapfel et al. (2015). In particular, our group has developed a constitutive relation for the modeling of arterial layers with a rotationally symmetric fiber dispersion (Gasser et al., 2006). Recently, this model has been extended to a more general case (Holzapfel et al., 2015) for which a non-symmetric fiber dispersion can also be captured.

Generally, the role of the fibers is primarily mechanical, providing the material with increased stiffness and strength. The fibers are elongated when loaded in tension, and it is often assumed that they do not contribute to the overall mechanical response of

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the material in compression. The computational implementation of this assumption requires a tension–compression 'switch' which eliminates the mechanical contribution of each fiber that is in compression. However, as pointed out in Holzapfel and Ogden (2015), such a condition has not been interpreted correctly in the literature and in finite element programs; see, for example, Abaqus 6.13 Analysis User's Guide (2013).

A Heaviside step function is sometimes introduced to eliminate the mechanical influence of the compressed fibers; see Ateshian et al. (2007, 2009); Federico and Gasser (2010) and Melnik et al. (2015). Theoretically, this method could successfully exclude the contribution of the compressed fibers from the total strain-energy function. However, as indicated in Federico and Gasser (2010), the presence of the Heaviside function renders the stress and elasticity tensors discontinuous. In the recent paper by Holzapfel and Ogden (2015) we have proposed a modified fiber dispersion model which incorporates a weighted strain-energy function that allows the exclusion of fibers under compression without the need for a Heaviside function. This model, which is based on the AI approach, was developed for planar and three-dimensional fiber dispersions and enables the stress and the elasticity tensors to be calculated in a straightforward way. However, the computational aspects of this modified model, specifically the form of the elasticity tensor and the integration boundary that admits only fibers which are extended, are not yet documented. Therefore, the aim of this study is to further develop this model for the purpose of computational implementation.

The present study is structured as follows. In Section 2 we present the continuum mechanical framework for the modified fiber dispersion model in a decoupled form suitable for finite element implementation, including the Cauchy stress and the elasticity tensors for both planar and three-dimensional fiber distributions. The boundary of the integration domain is also discussed for different deformation states. In Section 3 we introduce an adaptive finite element integration scheme for the numerical integration required for the stress and the elasticity tensors in the appropriate domain. In Section 4 the theory introduced in Section 2 is applied to several examples using the finite element scheme from Section 3. In particular, six representative numerical simulations are presented with the aim of demonstrating the efficacy of the proposed computational method. Finally, Section 5 summarizes the developed method and discusses possible future developments of the present study.

2. Continuum mechanical framework

In this section we outline the basic notation and fundamental results of nonlinear continuum mechanics in order to establish the mathematical description of fiber dispersion models, including the corresponding Cauchy stress and elasticity tensors. In particular, the integration boundary in the deformation space within which fibers are extended is also introduced.

2.1. Kinematics

Let $\mathscr{B}_0$ be a (stress-free) reference configuration of a continuum body and $\mathscr{B}$ its deformed configuration. The deformation map $\boldsymbol{\chi}(\mathbf{X})$ transforms a material point $\mathbf{X}\in\mathscr{B}_0$ into a spatial point $\mathbf{x}\in\mathscr{B}$. With this deformation map we define the deformation gradient $\mathbf{F}(\mathbf{X}) = \partial\boldsymbol{\chi}(\mathbf{X})/\partial\mathbf{X}$ and its determinant $J = \det \mathbf{F}(\mathbf{X})$, where J is the local volume ratio; we require $J > 0$.

Following the multiplicative decomposition of the deformation gradient in Flory (1961) and Ogden (1978) we decouple $\mathbf{F}$ into a spherical (dilatational) part $J^{1/3}\mathbf{I}$ and a unimodular (distortional) part $\overline{\mathbf{F}} = J^{-1/3}\mathbf{F}$, with $\det \overline{\mathbf{F}} \equiv 1$. We define the right Cauchy–Green tensor $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ and its modified counterpart $\overline{\mathbf{C}} = \overline{\mathbf{F}}^{\mathrm{T}}\overline{\mathbf{F}}$, respectively, with the related invariants $I_1 = \mathrm{tr}\, \mathbf{C}$ and $\bar{I}_1 = \mathrm{tr}\, \overline{\mathbf{C}}$.

2.2. Planar fiber dispersion model

The modified fiber dispersion model (Holzapfel and Ogden, 2015) that accounts only for fibers under extension requires numerical integration in the sub-domain of a unit sphere for which the fiber stretch is greater than one. For some soft biological tissues such as arterial walls the fiber dispersion in the thickness direction is smaller than in the in-plane direction (Schriefl et al., 2012), and for our present purposes we neglect the out-of-plane dispersion. We treat the material as incompressible, elastic and fiber-reinforced with a locally planar fiber dispersion. Without loss of generality we choose the thickness direction in such a material as the $\mathbf{E}_3$ Cartesian axis. Hence, an arbitrary in-plane fiber direction within a dispersion about a mean fiber direction $\mathbf{M}$ may be described by a unit vector $\mathbf{N}$ in the reference configuration as

$$\mathbf{N}(\Theta) = \cos\Theta\mathbf{E}_1 + \sin\Theta\mathbf{E}_2, \tag{1}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are the in-plane unit rectangular Cartesian basis vectors, and Θ is the angle between the fiber direction $\mathbf{N}$ and $\mathbf{E}_1$, as shown in Fig. 1. Also shown in Fig. 1 is the mean fiber direction $\mathbf{M}$ and the angle Θ_{M} that it makes with the $\mathbf{E}_1$ direction. Analogously to (1) we may write

$$\mathbf{M} = \cos\Theta_{\mathrm{M}}\mathbf{E}_1 + \sin\Theta_{\mathrm{M}}\mathbf{E}_2 \tag{2}$$

in the reference configuration, where Θ_{M} is a constant.

Since we are considering elastic materials, we assume that there exists a strain-energy function $\Psi(\mathbf{C},\{\mathbf{N}\})$, where $\{\mathbf{N}\}$ implies the dependence on the distribution of $\mathbf{N}$, that depends on the macroscopic deformation through $\mathbf{C}$, the underlying material structure through each direction $\mathbf{N}$, and a PDF $\rho(\Theta)$ that describes the fiber alignment and dispersion. For computational purposes, we assume that the strain-energy function can be decoupled as (Holzapfel, 2000)

$$\Psi(\mathbf{C},\{\mathbf{N}\}) = \Psi_{\mathrm{vol}}(J) + \Psi_{\mathrm{iso}}\left(\overline{\mathbf{C}},\{\mathbf{N}\}\right), \tag{3}$$

where the function Ψ_{vol} is a purely volumetric contribution while Ψ_{iso} represents the energy contribution of an isochoric (volume preserving) deformation through $\overline{\mathbf{C}}$. Suppose now the total isochoric strain-energy function Ψ_{iso} is the superposition of the energies contributed by the (non-collagenous) ground matrix and the collagen fibers, i.e. (Holzapfel et al., 2000)

$$\Psi_{\mathrm{iso}} = \Psi_{\mathrm{g}}\left(\overline{\mathbf{C}}\right) + \Psi_{\mathrm{f}}\left(\overline{\mathbf{C}},\{\mathbf{N}\}\right). \tag{4}$$

Following Holzapfel and Ogden (2015) and Holzapfel et al. (2000) we model the ground matrix as a neo-Hookean material $\Psi_{\mathrm{g}}(\bar{I}_1) = \mu(\bar{I}_1 - 3)/2$, where the parameter μ is the shear modulus in the reference configuration. The isochoric strain energy contributed by the fibers per unit reference volume associated with the direction $\mathbf{N}$ is assumed to be a function of the fiber stretch only. Thus, we adopt a modified form of the standard fiber reinforcing model (Qiu and Pence, 1997) for the contribution of a fiber along $\mathbf{N}$ in which $\bar{I}_4$ is used instead of I_4. This is given by

$$\Psi_{\mathrm{n}}\left(\bar{I}_4(\mathbf{N})\right) = \frac{\nu}{2}\left(\bar{I}_4(\mathbf{N}) - 1\right)^2, \tag{5}$$

where ν is a non-negative material constant with the dimension of stress and the modified fourth invariant is