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Coupled higher-order layerwise mechanics and finite element for cylindrical composite and sandwich shells with piezoelectric transducers

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Coupled higher-order layerwise piezoelectric laminate mechanics are presented, applicable to shallow cylindrical composite and sandwich shells subjected to static mechanical loads and/or electric voltages. The current formulation enables efficient prediction of (i) global electromechanical response, (ii) local through-thickness distribution of electromechanical variables and (iii) interlaminar shear stress at the interface between adjacent material layers. Using the developed mechanics, the effects of curvature, thickness and ply angle on the global and local through-thickness response of sandwich composite shells are studied.

1. Introduction

Sandwich composite shell structures with piezoelectric transducers combine excellent mechanical properties, such as high flexural stiffness to mass ratio, with tailoring options provided by composite face layers by means of fiber orientation and lamination, and monitoring/real-time control capabilities of the piezoelectric materials. In this kind of structures, the high thickness and strong inhomogeneity through the thickness may lead to delamination between piezoelectric and composite material layer. Thus, prediction of interfacial stress is crucial in the design stage, whereas its determination via in-service monitoring is essential in order to estimate if damage is about to occur. In the last twenty five years layerwise laminate theories have been proved to be an accurate tool for the estimation of through-thickness stress distributions in composite and sandwich composite shell structures (Reddy, 2004).

Early works on laminate modelling of composite shells with piezoelectric transducers have been extensively reviewed by Saravanos and Heyliger (1999) and Benjeddou (2000), while recent literature reviews have been conducted by Qatu et al. (2010) and Carrera et al. (2011). Coupled piezoelectric shell theories based on single-layer through-thickness kinematics have been developed, among others, by Tzou (1993), Lammering (1991) and Saravanos (1997), and more recently by Zemcik et al. (2007) and Legner et al. (2013). However, single-layer theories fail to predict the through-thickness distribution of interlaminar shear stress in composite and sandwich shells of arbitrary lamination and thickness. In order to improve such predictions, shell finite elements resting on coupled linear layerwise piezoelectric laminate theories have been reported by Tzou and Ye (1996) and Heyliger et al. (1996). In these theories the displacements and electric potential are assumed to vary linearly through the thickness of each discrete layer of the laminate. Thus, in order to capture piecewise higher-order profiles occurring through the thickness of the shell, a large number of discrete layers is required, which increases computational cost. Moreover, the value of transverse shear stress at the interface between composite-piezoelectric material can be only approximated, since its assumed variation in each layer is constant. An alternative to the linear layerwise shell theories are 3-D or 2-D piezo-elasticity solutions (Kapuria et al., 1997, Alibeigloo and Chen, 2010, Wu and Tsai, 2012, Kulikov and Plotnikova, 2014, Zhang et al., 2014), compared to which, finite element solutions appear to be more flexible in terms of considering design changes in geometry, boundary conditions, inclusion of patch transducers etc. Another alternative is to super-impose higher-order variations of displacements, globally smeared through the laminate thickness, on the linear layerwise displacement field, as reported by Oh and Cho (2007) and Nath and Kapuria (2009). Balamurugan and Narayanan (2009) combined classical laminate plate theory

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kinematics with layerwise trigonometric functions to approximate the in-plane displacements through the thickness of composite shells, while assuming quadratic variation of electric potential through the thickness of the piezoelectric layers. Recently, Yasin and Kapuria (2014) reported a layerwise C^0 continuous shell finite element for shallow sandwich shells with piezoelectric layers by superimposing a global through-thickness higher-order displacement field on linear layerwise distributions and a quadratic approximation of the electric potential through the thickness of the piezoelectric layers. Up to date, most existing laminate theories applicable to composite and sandwich shallow shells with piezoelectric transducers are either based on linear layerwise kinematics or on combination of these with globally smeared higher-order distributions through the thickness of the shell laminate.

In the present paper, a novel higher-order layerwise theoretical framework and a corresponding finite element are presented, capable of predicting the static electromechanical response of composite and sandwich composite cylindrical shells with piezoelectric transducers. Simple kinematic assumptions enable prediction of through-thickness piecewise distributions up to 3rd order using a low number of discrete layers, and prediction of stress at the interface between adjacent discrete layers. Validation studies with exact and Ritz type solutions based on single-layer and smeared higher-order layerwise formulations reveal the enhanced capabilities of the present shell laminate theory. Using the developed formulation, the effect of curvature, thickness and ply-angle on the electromechanical response of shallow cylindrical sandwich shell structures are quantified.

2. Piezoelectric cylindrical shell mechanics

The next paragraphs describe the theoretical formulation, starting from the governing material equations at the piezoelectric ply and arriving to the solution of the coupled electromechanical structural system of the sandwich composite cylindrical shell in sensory and/or active configuration.

2.1. Governing material equations

In general, the laminate layers including, piezoelectric, composite and foam plies are assumed to exhibit linear piezoelectric behaviour. Piezoelectric transducers polarized through-thickness are considered. The ply constitutive equations in the curvilinear coordinate system $O_1\xi\eta\zeta$ (Fig. 1(a) and (b)) have the form:

$$\begin{aligned} \boldsymbol{\sigma}_i &= C^E_{ij}\mathbf{S}_j - (e_{3i})^T\mathbf{E}_3 \\ \mathbf{D}_3 &= e_{3j}\mathbf{S}_j + \varepsilon^S_{33}\mathbf{E}_3 \end{aligned} \tag{1}$$

where i, j = 1, 2, 4, 5, 6, since transverse normal strain and stress (i, j = 3) are not considered, as indicated by the kinematic assumptions (3); $\boldsymbol{\sigma}_i$ and $\mathbf{S}_j$ are the mechanical stress and engineering strain, respectively, in vectorial notation; $\mathbf{E}_3$ is the electric field vector; $\mathbf{D}_3$ is the electric displacement vector; C_{ij} is the elastic stiffness tensor; e_{3j} is the piezoelectric tensor arising from the piezoelectric charge tensor and the stiffness tensor; and ε_{33} is the electric permittivity tensor of the material. Superscripts E and S indicate a constant electric field, and strain conditions, respectively. The electric field vector $\mathbf{E}_3$ is given by:

$$\mathbf{E}_3 = -\partial\boldsymbol{\varphi}/\partial\zeta \tag{2}$$

where $\boldsymbol{\varphi}$ indicates electric potential along the thickness direction of the piezoelectric transducer.

2.2. Through-thickness kinematic assumptions

A section of a typical cylindrical laminate is shown in Fig. 1(a). It is assumed to be subdivided into n discrete layers as shown schematically in Fig. 1(c). Each discrete-layer may contain a single ply, a sub-laminate, or a sub-ply. Parabolic and cubic variations of in-plane displacement and electric potential through the thickness of each discrete layer are superimposed on linear respective approximations (Fig. 1(d)). In this context, the variation of displacement and electric potential in the k-th discrete layer take the form:

$$\begin{aligned} \mathbf{u}^k(\xi,\eta,\zeta_k) &= U^k(\xi,\eta)\Psi^k_1(\zeta_k) + U^{k+1}(\xi,\eta)\Psi^k_2(\zeta_k) + \alpha^k_\xi(\xi,\eta)\Psi^k_3(\zeta_k) + \lambda^k_\xi(\xi,\eta)\Psi^k_4(\zeta_k) \\ \mathbf{v}^k(\xi,\eta,\zeta_k) &= V^k(\xi,\eta)\Psi^k_1(\zeta_k) + V^{k+1}(\xi,\eta)\Psi^k_2(\zeta_k) + \alpha^k_\eta(\xi,\eta)\Psi^k_3(\zeta_k) + \lambda^k_\eta(\xi,\eta)\Psi^k_4(\zeta_k) \\ \mathbf{w}^k(\xi,\eta,\zeta_k) &= w^0(\xi,\eta) \\ \boldsymbol{\varphi}(\xi,\eta,\zeta_k) &= \Phi^k(\xi,\eta)\Psi^k_1(\zeta_k) + \Phi^{k+1}(\xi,\eta)\Psi^k_2(\zeta_k) + \alpha^k_\varphi(\xi,\eta)\Psi^k_3(\zeta_k) + \lambda^k_\varphi(\xi,\eta)\Psi^k_4(\zeta_k) \end{aligned} \tag{3}$$

where superscripts k = 1, … , n denote discrete layer and ζ_k is the local thickness coordinate of layer k defined such as $\zeta_k = 0$ at the middle of the discrete layer, $\zeta_k = 1$ and $\zeta_k = -1$ at the top and the bottom, respectively. Ψ^k_1, Ψ^k_2 are linear and Ψ^k_3, Ψ^k_4 are quadratic, cubic interpolation functions, respectively, through the thickness of the layer (Appendix A).

The first two terms on the right hand side of the approximations of the in-plane displacements and electric potential in equation (3) describe the linear field, and U^k, V^k, U^{k+1}, V^{k+1} and Φ^k, Φ^{k+1} are the respective values at bottom and top of the discrete layer, effectively describing extension and rotation, and electric potential at the terminals, respectively, of the layer. The last two terms describe quadratic and cubic variations of displacements and electric potential through the thickness of the discrete layer and vanish at its top and bottom interfaces, since the polynomial functions Ψ^k_3 and Ψ^k_4 ensure displacement/potential continuity across the discrete layer boundaries. The terms $\alpha^k_\xi, \alpha^k_\eta$ and $\lambda^k_\xi, \lambda^k_\eta$ are higher-order elastic and $\alpha^k_\varphi, \lambda^k_\varphi$ electric degrees of freedom of each discrete layer introduced by the quadratic and cubic polynomials, respectively. The present model distinguishes in this point from higher-order approximations globally smeared through the laminate thickness, since the higher-order terms are additional degrees of freedom of the discrete layer.

2.3. Strain–displacement relations

In the case of a shallow cylindrical shell (h/R << 1 (Soedel, 2004), where R is the radius of curvature of the shell and h is the