European Journal of Mechanics A/Solids 54 (2015) 30–43

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Thermal postbuckling analysis of anisotropic laminated beams with different boundary conditions resting on two-parameter elastic foundations

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ARTICLE INFO

Article history:
Received 28 June 2014
Accepted 1 June 2015
Available online 23 June 2015

Keywords:
Anisotropic laminated composite beams
Higher-order shear deformation beam theory
Thermal postbuckling

ABSTRACT

Thermal postbuckling analysis of shear deformable anisotropic laminated composite beams with temperature-dependent material properties subjected to uniform temperature distribution through the thickness and resting on a two-parameter elastic foundation is presented. The material of each layer of the beam is assumed to be linearly elastic and fiber-reinforced. The governing equations are based on Reddy's high order shear deformation beam theory with a von Kármán-type of kinematic nonlinearity. Composite beams with clamped–clamped, clamped–hinged, and hinged–hinged boundary conditions are considered. A numerical solution for the nonlinear partial-integral differential form in terms of the transverse deflection using Galerkin's method is employed to determine the buckling temperatures and postbuckling equilibrium paths of anisotropic laminated beams with uniform temperature distribution through the thickness. The numerical illustrations on the thermal postbuckling response of laminated beams with different types of boundary conditions, ply arrangements (lay-ups), geometric and physical properties are also presented, and the results reveal that the geometric and physical properties, temperature dependent properties, boundary conditions, and elastic foundation all have a significant effect on thermal postbuckling behavior of anisotropic laminated composite beams.

1. Introduction

Composite structures, like beams, are widely used in various engineering applications such as airplane wings, helicopter blades as well as many others in aerospace, mechanical, and civil industries. Anisotropic composites provide more design flexibility than conventional materials. Due to their outstanding engineering properties, such as high strength/stiffness to weight ratios, the laminated composite beams are likely to play a remarkable role in the design of various engineering structures and partially replace the conventional isotropic beam structures. However, their increased amount of design parameters and physical phenomena brings some difficulties in analysis of aerospace vehicles, supersonic/hypersonic flight vehicles, reusable space transportation systems, etc. One of the important problems in engineering structures is the thermal buckling of composite beams.

A number of useful elastic beams, plates and shells theories have been proposed for analysis of composite structures (Levy, 1877; Reissner, 1975; Levinson, 1981; Murty, 1984; Reddy, 1984; Stein, 1986; Looss and Joseph, 1990; Touratier, 1991; Wu and Chen, 2008; Kargani et al., 2013; Shen, 2011, 2013; Esfahani et al., 2013; Shen and Wang, 2014) due to the rapidly increasing use of advanced composite materials in various industries. The importance and potential benefits of composite beams in engineering practice have inspired continuing research interest. As it is well known that the buckling problems are the geometrically nonlinear ones, the temperature rise causes compressive forces in the beams with immovable ends, therefore leading to the occurrence of buckling phenomenon. A series of research work in the thermal postbuckling analysis of composite beams have been

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http://dx.doi.org/10.1016/j.euromechsol.2015.06.001
0997-7538/

conducted in the past decades. Kaju and Kao (1984) investigated the thermal postbuckling of columns. Gauss and Antman (1984) presented the global descriptions of the properties of buckled states of nonlinear thermoelastic beams and plates when heated at their ends and edges. Jekot (1996) evaluated the thermal postbuckling of a beam made of physically nonlinear thermoelastic material using the geometric equations in the von Kármán strain-displacement approximation. Sapountzakis and Tsiatas (2007) studied the flexural buckling of composite Euler-Bernoulli beams of arbitrary cross sections, in which the resulting boundary-value problems were solved using the boundary element method. Nayfeh and Emam (2008) investigated the postbuckling behavior of beams composed of isotropic materials under the axial load with various boundary conditions. Lee and Choi (1999) studied the thermal buckling and postbuckling behaviors of a composite beam with embedded shape memory alloy (SMA) wires using the finite element model in which the SMA wire actuators are modeled with the beam elements and the composite laminates with the shell elements. Asadi et al. (2013, 2014) investigated the large amplitude vibration and nonlinear buckling of SMA fiber-reinforced hybrid composite beams with symmetric and asymmetric lay-up using the Euler-Bernoulli beam theory and the nonlinear von-Karman strain field. When the Euler–Bernoulli beam theory is used for the buckling analysis of laminated beams, the buckling loads are overestimated, due to the consequence of neglecting the transverse shear strain and assuming that the plane normal to the beam axis still remains normal even after the deformation. Due to the high ratio of the extensional modulus to the transverse shear modulus of composite beams, the shear deformation cannot be ignored even for reasonably large slenderness ratio. The first-order shear deformation beam theory (FSDBT), such as Timoshenko beam theory, is then used to describe the kinematics of deformation of laminated beams accurately because the transverse shear stresses are accounted for. Abramovich (1994) investigated the thermal buckling of cross-ply symmetric and nonsymmetric composite beams to solve Timoshenko type equations. Bert (1973) and Dharmarajan and McCutchen (1973) obtained the shear correction coefficients for orthotropic beams. Vosoughi et al. (2012) investigated the thermal buckling and postbuckling responses of symmetric laminated composite beams with temperature-dependent material properties, in which the governing equations are based on the FSDBT with the von Kármán assumptions. They obtained the critical temperature as well as the nonlinear equilibrium path (postbuckling behavior) of symmetric laminated beams using a direct iterative method. In fact, the variation in material properties along the thickness of unsymmetrically laminated plates or beams results in quite a different behavior compared to that of symmetrically laminated or single material plates or beams. For example, the bifurcation buckling cannot generally occur for unsymmetrically laminated plates or beams with simply supported edges due to in-plane loadings, i.e., a transverse deflection is initiated, regardless of the magnitude of the loading, which is often the case with laminated composite materials (Leissa, 1986, 1987; Qatu and Leissa, 1993). Khdeir and Reddy (1997) studied the buckling behaviors of cross-ply laminated beams with arbitrary boundary conditions based on the classical, first-, second- and third-order beam theories. Khdeir (2001) evaluated the thermal buckling behavior of cross-ply laminated beams subjected to a uniform temperature rise for various boundary conditions and obtained the exact solution for the critical buckling temperature. Akba and Kocatürk (2011) presented the postbuckling analysis of a simply supported beam subjected to a uniform thermal loading using a total Lagrangian finite element model of two dimensional continuum for an eight-node quadratic element.

Patel et al. (1999) studied the free vibration and post-buckling analysis of laminated orthotropic beams resting on a two parameters elastic foundation (Pasternak type) using a three-node shear flexible beam element. Based on shear deformable beam theory, Aydogdu (2007) conducted the thermal buckling analysis of cross-ply laminated beams subjected to different sets of boundary conditions by the Ritz method. Ghugal and Shimpi (2001) presented a number of beam theories in the literature as well as a review of displacement- and stress-based refined theories for isotropic and anisotropic laminated beams. They solved and evaluated the nonlinear governing equation for the thermal postbuckling of the composite beam using the nonlinear finite element formulation.

In spite of the availability of FEM and powerful computer programs, the second- or higher-order analysis of a composite beam is still an impractical task to most structural designers due to the limitation of the number of degrees of freedom (DOF) required to achieve a desired level of precision and efficiency. Recently, comparing the modern approaches to the classical beam theories, including well-known classical results related to Euler-Bernoulli and Timoshenko beam theories, Carrera and Giunta (2010), Carrera and Petrolo (2011), Carrera et al. (2010a,b, 2011a,b) and Giunta et al. (2010, 2011) established the Carrera Unified Formulation (CUF) which has hierarchical properties and is capable of dealing with most typical engineering challenges; in particular, the error can be reduced by increasing the number of the unknown variables. It overcomes the problem of classical formulae that require different formulas for tension, bending, shear and torsion. More important, it can be applied to any beam geometries and loading conditions, reaching a high level of accuracy with low computational cost, and it can tackle problems which in most cases are solved by employing the plate/shell and 3D formulations. As expected, their work provided an effective means in applications related to bridge structures, aircraft wings, helicopters and propeller blades. Vaz et al., 2007 examined a perturbation solution for the initial postbuckling of beams that were supported on an elastic foundation under uniform thermal load. Emama and Nayfeh (2009) obtained a closed-form solution for the postbuckling configurations of composite beams with various boundary conditions based on the classical beam theory and expressed these configurations as functions of the applied axial load. Gupta et al. (2010) studied the thermal postbuckling of columns with axially immovable ends with the Rayleigh–Ritz method. Kim et al. (2013) developed the refined and accurate laminated composite beam element based on the eigenvalue problem for the flexural and torsional analyses. Recently, Ma and Lee (2011, 2012) presented an excellent result of geometrically nonlinear static responses of functionally graded materials (FGM) beams subjected to a uniform in-plane thermal loading using the first-order shear deformation beam theory. They stated that the FGM beams with different boundary conditions have some inherent characteristics due to the inhomogeneous boundary conditions. To the authors' best knowledge, there is no work available in the literature on the thermal postbuckling analysis of shear deformable anisotropic laminated composite beams with different types of boundary conditions resting on two-parameter nonlinear elastic foundation.

The present work focuses on the thermal postbuckling analysis of anisotropic laminated composite beams with temperature-dependent material properties subjected to uniform temperature distribution through the thickness and resting on two-parameter (Pasternak-type or Vlasov-type) elastic foundation. The governing equations are based on Reddy's high order shear deformation beam theory (Reddy, 2004) with a von Kármán-type of kinematic nonlinearity including the beam–foundation interaction. The analysis uses a Galerkin's method to determine the thermal buckling temperatures and postbuckling equilibrium paths of a beam