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Application of the coupled stress-energy criterion to predict the fracture behaviour of layered ceramics designed with internal compressive stresses

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ABSTRACT

One novel approach to improve the apparent toughness of ceramics is to design a multilayer architecture with embedded layers having compressive residual stresses. Surface cracks propagating during mechanical loading can be deflected within the compressive layers, in order to delay the final fracture of the whole structure. The design of high toughness laminates requires understanding the effect of residual stresses on the initiation and propagation of cracks in the material.

In this work, a coupled stress-energy criterion is used to predict the initiation and propagation of surface cracks in ceramic laminates upon thermo-mechanical loading. Experiments were conducted on V-notched alumina-based laminates to show the effect of residual stresses and mechanical loading on their fracture behaviour. The conditions for crack initiation as predicted for notched specimens agreed with the experimental observations. It is shown that the onset of cracks from V-notches is associated with (i) the tensile residual stresses in the first surface layer and (ii) the depth of the notch. The further propagation of the crack into the first embedded compressive layer was also studied. Based upon the coupled criterion, a short penetration of the propagating crack into the first compressive is foreseen. If the mechanical load is increased, the crack finally deflects within the compressive layer propagating with a certain angle which is also predicted with a good accuracy.

1. Introduction

A limitation for the use of ceramics is their low fracture toughness, which often causes spontaneous brittle failure of the component or system. Contrary to metals, crack propagation in brittle materials such as ceramics is usually catastrophic, due to the lack of plastic deformation. The brittle fracture of ceramics is a consequence of the material defects located either within the bulk or especially at the surface, resulting from the processing and/or machining procedures (Morrell, 1999; Danzer, 2002). Under external applied stress, the stress concentration associated with such defects is the driving force for crack propagation, causing the failure of ceramic components.

Increasing strength in ceramics can be attained by reducing the size of these critical defects (e.g. through colloidal processing) (Lange, 1989), and introducing compressive residual stresses at the surface (e.g. strengthening in glass such as Gorilla glass (Corning, 2012)). However, significant reduction of strength variability cannot be achieved with these approaches. In recent years, a "flaw-tolerant" approach has emerged for building tougher ceramics in a "bio-inspired" layered architecture, combining materials with different microstructures, or properties similar to those found in nature (Launey and Ritchie, 2009). Composite materials using such symmetric multilayer architectures (*e.g.* ceramic composites such as alumina–zirconia and mullite–alumina among others) have been reported to exhibit increased fracture toughness, higher energy absorption capability and/or non-catastrophic fracture behaviour in comparison to their constituent (monolithic) materials. Among the various laminate designs reported in the literature, two main approaches regarding the fracture energy of the layer

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interfaces must be highlighted, i.e. the use of “weak” or “strong” interfaces. A particular case of the latter is based on the capability of inducing residual stresses in the layers during cooling from sintering, in order to provide a barrier to crack propagation and, in some cases, even stop cracks (Rao and Lange, 2002). Then, understanding crack propagation in layered ceramic is necessary to optimize their mechanical behaviour. A key feature is the contribution of the residual stresses to the fracture toughness of the individual layers.

The experimental evaluation of the crack growth resistance in brittle materials is often performed on single edge notched (or V-notched) specimens loaded under bending, so called SENB or SEVNB test (Damani et al., 1996). This requires the introduction of a crack (or sharp notch), which is in many cases a challenge in brittle materials. The fracture criterion of brittle materials is usually described by linear elastic fracture mechanics (LEFM), based on the Griffith/Irwin law (Griffith, 1921; Irwin, 1962) but is ineffective in predicting the initiation of a new crack, especially emanating from a notch.

An alternative approach to predict the initiation and propagation of surface cracks in ceramic laminates upon thermo-mechanical loading is to use a coupled stress-energy criterion (Leguillon, 2002; Leguillon et al., 2015). It was developed over the last decade within a more general framework baptized Finite Fracture Mechanics (Leguillon, 2002; Martin and Leguillon, 2004; Taylor et al., 2005; Cornetti et al., 2006, 2012; Yosibash, 2012). This criterion states that crack onset occurs if two necessary conditions are fulfilled simultaneously: the first one specifies that there is enough available energy to create a crack and the second that the tensile stress is greater than the tensile strength all along the expected crack path. As a consequence of the energy balance (i.e. the first condition), the crack nucleation occurs abruptly, the crack jumps over a given length. This length is not an adjustable parameter, but a direct consequence of the two conditions: one providing a lower bound for admissible crack lengths and the other giving an upper bound. The compatibility between these two bounds is obtained if the load is sufficiently high.

In this work, the conditions for crack initiation as predicted for notched specimens are studied and compared with experimental observations on notched ceramic laminates under thermo-mechanical loading. Particular attention is put in the description of crack propagation and deflection into the first embedded compressive layer.

2. The tested specimens and the failure mechanisms

The specimens investigated here were made by sequential slip casting (Bermejo et al., 2006, 2007), consisting of a stacking sequence of 9 alternated layers of two different ceramic materials: 4 thin layers of Al_2O_3 with 30% monoclinic ZrO_2 (referred to as AMZ layers), sandwiched between 5 thicker layers of Al_2O_3 with 5% tetragonal ZrO_2 (named ATZ layers). Different laminate samples were obtained by varying the volume ratio V_R between ATZ and AMZ material, ranging between Vol. (ATZ)/Vol. (AMZ) ≈ 6 to 10. It is worthy pointing out that, as a consequence of the processing route employed (i.e. slip casting), slightly differences in the thickness of a particular layer material may occur. Table 1 shows the thicknesses of the ATZ and AMZ layers of four samples selected for this investigation. Sample P0 is extracted from previous data (Bermejo et al., 2006; Sevecek et al., 2013) and samples P1, P2, P3 belong to a new series of tests dedicated to the present investigation.

Table 2
E is the Young modulus, ν the Poisson ratio, α the coefficient of thermal expansion, K_{Ic} the material toughness and G_c the fracture energy.

Material	E (GPa)	ν	α (K^{-1})	σ_c (MPa)	K_{Ic} (MPa $m^{1/2}$)	G_c (J m^{-2})
ATZ (1)	390	0.22	9.8 10^{-6}	422	3.2	25
AMZ (2)	280	0.22	8.0 10^{-6}	90	2.6	23

The elastic and fracture parameters of the corresponding ATZ and AMZ layers, measured in monolithic samples, are those given in (Ševeček et al., 2013) and are summarized in Table 2.

The fracture energy G_c relies on K_{Ic} (Table 1) through the Irwin formula (under the plane strain assumption)

$$G_c^{(i)} = \frac{1-\nu^{(i)2}}{E^{(i)}} K_{Ic}^{(i)2} \quad i = 1, 2 \tag{1}$$

Here and in the following, the upper index $^{(1)}$ holds for ATZ and $^{(2)}$ for AMZ. It may be noted that the relationship (1) allows calling interchangeably $G_c^{(i)}$ and $K_{Ic}^{(i)}$ as the toughness of material $^{(i)}$.

The geometry used in the modelling is shown in Fig. 1. The ATZ layers thickness was taken as t_1 =0.7 mm and that of AMZ as t_2 =0.14 mm. It can be noted that the thicknesses in the model geometry are averaged from sample P0 and are close to sample P3.

The sintering of the laminate samples requires firing the green stacks at 1550 °C and then cooled down slowly to room temperature (see Bermejo et al., 2006 for more details). Due to the change of temperature $\Delta\theta$ and the mismatch in the coefficients of thermal expansion $\alpha^{(i)}$, in-plane bi-axial residual stresses are induced inside the layers. It is assumed that no plastic deformation occurs which is quite likely in this type of material, and the stresses can be easily calculated using the following formula

$$\begin{cases} \sigma_R^{(1)} = E^{(1)*} \dfrac{\left(\alpha^{(2)} - \alpha^{(1)}\right)\Delta\theta}{\dfrac{E^{(1)*}}{E^{(2)*}} V_R + 1} \\ \sigma_R^{(2)} = -E^{(2)*} \dfrac{\left(\alpha^{(2)} - \alpha^{(1)}\right)\Delta\theta}{\dfrac{E^{(2)*}}{E^{(1)*}} \dfrac{1}{V_R} + 1} \quad \text{with } E^{(i)*} = \dfrac{E^{(i)}}{1-\nu^{(i)}} \quad \text{for } i = 1, 2 \end{cases} \tag{2}$$

Table 1
Thicknesses of the ATZ and AMZ layers in 4 different samples. The volume ratio V_R is defined as Vol. (ATZ)/Vol. (AMZ). Sample P0 is extracted from previous data (Bermejo et al., 2006; Sevecek et al., 2013) and samples P1, P2, P3 belong to a new series of tests.

Sample	ATZ thicknesses (mm)					AMZ thicknesses (mm)				V_R
	L1	L3	L5	L7	L9	L2	L4	L6	L8	
P0	0.773	0.616	0.623	0.634	0.819	0.147	0.125	0.142	0.148	6.17
P1	0.590	0.575	0.578	0.600	0.580	0.100	0.104	0.104	0.109	7.01
P2	0.680	0.555	0.525	0.523	0.500	0.073	0.075	0.074	0.065	9.70
P3	0.830	0.654	0.642	0.661	0.668	0.141	0.144	0.141	0.149	6.01