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Comparison of two models for anisotropic hardening and yield surface evolution in bcc sheet steels

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The purpose of the current work is the investigation and comparison of aspects of the material behavior predicted by two models for anisotropic, and in particular cross, hardening in bcc sheet steels subject to non-proportional loading. The first model is the modified form (Wang et al., 2008) of that due to Teodosiu and Hu (1995, 1998). In this (modified) Teodosiu-Hu model (THM), cross hardening is assumed to affect the yield stress and the saturation value of the back stress. The second model is due to Levkovitch and Svendsen (2007) and Noman et al. (2010). In the Levkovitch-Svendsen model (LSM), cross hardening is assumed to affect the flow anisotropy. As clearly demonstrated in a number of works applying the THM (e.g., Boers et al., 2010; Bouvier et al., 2005, 2003; Hiwatashi et al., 1997; Li et al., 2003; Thuillier et al., 2010; Wang et al., 2008) and the LSM (e.g., Clausmeyer et al., 2014, 2011b; Noman et al., 2010), both of these are capable of predicting the effect of cross hardening on the stress-deformation behavior observed experimentally in sheet steels. As shown in the current work, however, these two models differ significantly in other aspects, in particular with respect to the development of the yield stress, the back stress, and the yield surface. For example, the THM predicts no change in the shape of the yield surface upon change of loading path, in contrast to the LSM and crystal plasticity modeling of bcc sheet steels (Peeters et al., 2002). On the other hand, the LSM predicts no hardening stagnation after cross hardening as observed in experiments, in contrast to the THM. Examples are given.

1. Introduction

Finite-element-based modeling and simulation of the material and structural behavior of sheet metal parts in various stages of design and manufacture is today standard. In general, one aim of this is to benefit from the predictive capability of such simulations (Zienkiewicz et al., 2010). In this regard, Wagoner et al. (2013) emphasize the importance of improving material models to account for the loading path-dependent behavior of metals during sheet metal forming. When subject to complex non-proportional loading processes such as those found in many technological applications, a number of metals exhibit hardening behavior which is more complex than isotropic and kinematic hardening alone. Observed effects in this regard include cross hardening and hardening stagnation during orthogonal loading (e.g., tension to shear). Cross hardening is observed to occur for example in a number of steels such as austenitic fcc tube steels (e.g., SUS304: Ishikawa, 1997; Wu, 2003), ferritic bcc tube steels (e.g., S355: Kowalewski and Sliwowski, 1997), multi-phase tube steels (e.g., X100: Shinohara et al., 2010), or ferritic bcc sheet steels (e.g., LH800: Ghosh and Backofen, 1973; Noman et al., 2010). Systematic studies (Bouvier et al., 2005, 2006a, 2003) of interstitial free (IF), high-strength low-alloyed (HSLA), transformation-induced plasticity (TRIP), and dual-phase (DP), sheet steels, found significant kinematic hardening, hardening stagnation, as well as cross hardening, the latter especially in IF sheet steels. In these investigations, the material was subjected to monotonic shear, reverse shear, as well as orthogonal tension-shear, loading. Clausmeyer et al. (2012); van Riel and van den Boogaard (2007); Wang et al. (2008) have documented these effects in the IF sheet steel DC06 with the help of monotonic tension, reverse shear, and orthogonal tension-shear,

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tests, all under plane-strain conditions. In particular, cross hardening occurs during discontinuous (e.g., tension-shear: Bouvier et al., 2005, 2006a, 2003) and continuous (e.g., tension-shear: Noman et al., 2010; van Riel and van den Boogaard, 2007; Wang et al., 2008) orthogonal tension-shear tests. Similar results were obtained by Verma et al. (2011) in a series of tension and compression tests on ultra-low carbon IF sheet steel in which the tension or compression direction changed from rolling to transverse. In these tests, cross hardening was correlated with a change of the tension axis. As attested to in particular by the continuous orthogonal tension-shear test results (Noman et al., 2010; van Riel and van den Boogaard, 2007; Wang et al., 2008), cross hardening is transient and strongly depends on the rate of transition. Its occurrence and strength are strongly influenced by the particular path taken in stress space in changing from one loading direction to another.

Generally speaking, anisotropic hardening in sheet steels may be influenced by the grain and dislocation (micro)structures. In particular, the former is related to the grain orientation distribution (texture). The influence of texture on the hardening behavior of IF sheet steel was investigated by Bacroix and Hu (1995) and Nesterova et al. (2001a,b) using two-stage loading tests (e.g., shear to reverse shear, tension to shear). In particular, Bacroix and Hu (1995) concluded that, at least up to "moderate" strains, the influence of texture *evolution* on hardening in the specimens investigated was small compared to that of dislocation structure evolution. This conclusion was substantiated by later crystal plasticity modeling (e.g., Peeters et al., 2002). Related to this are more recent EBSD investigations on DC06 (Boers et al., 2010; Clausmeyer et al., 2012), which imply that the rolling-induced texture in this steel does not change considerably for strains lower than 35% in simple tension. This may be the case in other ferritic steels (e.g., LH800: Clausmeyer et al., 2012; Noman et al., 2010) as well. These results imply that it is sufficient to account for the effect of the initial (e.g., rolling) texture on the anisotropic hardening and flow behavior in the material model.

Although texture evolution in this sense may be secondary, grain orientation (i.e., glide-system orientation) in relation to loading direction certainly influences dislocation structure development. The development of certain characteristic dislocation structures related to cross hardening have been observed during quasi-static loading of mild steels such as DC06 at room temperature (e.g., Rauch and Schmitt, 1989; Rauch and Thuillier, 1993; Thuillier and Rauch, 1994). These include for example dense dislocation wall structures. The morphology and orientation of such walls depends for example on grain orientation, the type of loading, and the loading direction in relation to the grain orientation (e.g., Clausmeyer et al., 2012; Nesterova et al., 2001a,b; Thuillier and Rauch, 1994). A change in loading direction or type activates new glide systems for which existing walls act initially as obstacles, resulting in cross hardening.

One of the first phenomenological models accounting in particular for cross hardening is the Teodosiu-Hu model (THM: e.g., Hu et al., 1992; Teodosiu and Hu, 1995, 1998). In the THM, cross hardening is assumed to affect the yield stress σ_{Y} in the yield function ϕ_{Y}. The THM has been employed in a number of works (e.g., Bouvier et al., 2005, 2003; Haddadi et al., 2006; Hiwatashi et al., 1997; Li et al., 2003; Thuillier et al., 2010) to model anisotropic flow and hardening behavior in sheet metals. This has motivated similar work on models for anisotropic hardening in the continuum (Barlat et al., 2013; Butuc et al., 2011; Carvalho Resende et al., 2013; Clausmeyer et al., 2014; Pietryga et al., 2012; Shi and Mosler, 2012; Tarigopula et al., 2008, 2009) and crystal plastic (Peeters et al., 2002; Viatkina et al., 2007) contexts. More recently, the THM has been modified, extended and generalized to deal with arbitrary changes of loading path by Wang et al. (2008). This modified version of the THM is that considered in the current work.

A second model for cross hardening was introduced by Levkovitch and Svendsen (2007) and Noman et al. (2010). This model has been referred to by Shi and Mosler (2012) as the "Levkovitch–Svendsen" model (LSM), who discussed related models for distortional hardening and the strength differential effect in magnesium alloys. In the LSM, cross hardening is assumed to influence the flow anisotropy through the corresponding tensor $\mathscr{A}$ determining ϕ_{Y}. Common to both the THM and the LSM is the constitutive form

$$\phi_{\mathrm{Y}} = \sqrt{(\boldsymbol{M}-\boldsymbol{X})\cdot\mathscr{A}(\boldsymbol{M}-\boldsymbol{X})} - \sigma_{\mathrm{Y}} \tag{1}$$

for ϕ_{Y} with respect to the intermediate (local) configuration. Here, $\boldsymbol{M}$ is the Mandel stress (e.g., Mandel, 1971, 1974), and $\boldsymbol{X}$ is the back stress. To be more precise, in the THM, σ_{Y} is assumed to depend on both isotropic and cross hardening, and $\mathscr{A}$ is assumed constant. On the other hand, in the LSM, cross hardening is assumed to influence the evolution of $\mathscr{A}$, and σ_{Y} is assumed to depend only on isotropic hardening. As shown in the previous works discussed above, both models are capable of quantitatively predicting experimentally observed cross hardening. The question arises as to how the THM and the LSM compare in other respects. To this end, in the current work, a direct comparison of these two has been carried out. To this end, both models have been identified from the same data set for the ferritic sheet steel DC06 (for comparison, the ferritic-pearlitic steel LH800 is also briefly discussed). As the current comparison of the THM and the LSM shows, the two models are not equivalent in other respects. Among these, the prediction of yield surface evolution is perhaps the most prominent.

The current work begins with a brief review of the formulation of the two models in Section 2. This is carried out within the framework of the multiplicative decomposition of the deformation gradient and the assumption of small elastic strain relevant to metal inelasticity. Again, for a meaningful comparison, the two models are identified in Section 3 using the same test data sets for the ferritic bcc sheet steel DC06. The identified THM and LSM are then compared on the basis of their respective predictions for yield and back stress evolution in Section 4 as well as yield surface development in Section 5. The latter results are also compared qualitatively with analogous results from the crystal plasticity model of Peeters et al. (2002) for IF steel. Lastly, these two models are compared in the context of their application to the modeling of sheet metal forming during non-proportional loading in Section 6. The work ends with a summary and discussion in Section 7.

2. Model formulation

2.1. Notation

In this work, Euclidean vectors (i.e., first-order Euclidean tensors) are represented by lower-case bold italic characters $\boldsymbol{a},\boldsymbol{b},\ldots$; in particular, let $\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3$ represent the Cartesian basis vectors. Likewise, upper-case bold italic characters $\boldsymbol{A},\boldsymbol{B},\ldots$ represent second-order Euclidean tensors; in particular, let $\boldsymbol{I}$ represent the second-order identity tensor. Such tensors are defined in this work as linear mappings between (three-dimensional) Euclidean vectors. In other words, $\boldsymbol{A}\boldsymbol{b}$ is a vector for all $\boldsymbol{A}$ and all $\boldsymbol{b}$. Let $\boldsymbol{I}\cdot\boldsymbol{A}$ and $\operatorname{dev}\boldsymbol{A} = \boldsymbol{A}-(\boldsymbol{I}\cdot\boldsymbol{A})\boldsymbol{I}/3$ represent the trace and deviatoric part, respectively, of any $\boldsymbol{A}$. Likewise, let $\operatorname{sym}\boldsymbol{A}:=(\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}})/2$ and $\operatorname{skw}\boldsymbol{A}:=(\boldsymbol{A}-\boldsymbol{A}^{\mathrm{T}})/2$ represent the symmetric and skew-symmetric parts, respectively, of any $\boldsymbol{A}$. Fourth-order tensors are represented by upper-case calligraphic characters $\mathscr{A},\mathscr{B},\ldots$, in this work. Interpreting these as linear mappings between second order tensors, $\mathscr{A}\boldsymbol{B}$ represents a second