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Vibration insight of a nonlocal viscoelastic coupled multi-nanorod system

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Longitudinal vibration of viscoelastic multi-nanorod system (VMNS) is studied. Based on the D' Alembert's principles, nonlocal and viscoelastic constitutive relations, the system of m partial differential equations are derived which described the motion of the presented nano-system. Clamped–Clamped and Clamped–Free boundary conditions and two different chain systems, namely "Clamped-Chain" and "Free-Chain" are illustrated. The method of separations of variables and trigonometric method are utilized for solutions. The analytical expressions for critical viscoelastic parameters and asymptotic frequencies are presented. The predicted results are validated with results obtained by direct numerical simulations and results from literature. The effects of nonlocal parameter, number of nanorods, viscoelastic material constant and parameter of viscoelastic layer on the complex eigenvalue are discussed in details.

1. Introduction

Recently, growing interest in the dynamic response of nanostructures elements like a nanorods, nanobeams or nanoplates, plays an important role in the development of nanodevices. Therefore, the issue of vibration behavior of nanostructures elements has become very important from the practical point of view and it has wide application in nanotechnology. The nanodevices include biosensors (Ziegler, 2004; Sotiropoulou and Chaniotakis, 2003; Wang, 2005; Wang et al., 2003; Shen et al., 2012; Ali et al., 2009; Chowdhury et al., 2011), mass sensors (Lee et al., 2010; Mehdipour et al., 2011; Murmu and Adhikari, 2011), nanoresonators (He et al., 2005; Liu et al., 2011), gas sensors (Basu and Bhattacharyya, 2012; Llobet, 2013), nanoopto-mechanical system (Hierold et al., 2007; Lu et al., 2007) etc. Nanomaterial's such as carbon nanotubes (CNTs) (Iijima, 07 November 1991), boron nitride nanotubes (BNNTs) (Chopra et al., 18 August 1995), zinc oxide nanotubes (ZnO) (Liu and Zeng, 2009) and graphene sheet (Geim and Novoselov, 2007) are the basis material of many nanostructures and nanodevices. These nanomaterial's have extraordinarily properties resulting from their nanoscale dimensions (Guz et al., 2007; Gouadec and Colomban, 2007; Kuo et al., 2005; Dresselhaus et al., 2004; Ruoff et al., 2003). Performing controlled experiments at the nano-level is very difficult and expensive. Therefore, development appropriate mathematical models based on Eringen's continuum theory, which takes into account size effect and atomic forces is very important. By ignoring these effects in the development of mathematical models of nanoscale structures can cause completely incorrect solutions and hence erroneous designs. According to a paper (Eringen and Edelen, 1972), Eringen derived a constitutive relation in integral form, based on the assumption that the stress at the point is function of the strain at all points of the elastic body. Since then, many researchers have contributed to the development of nonlocal continuum theory and application in mathematical modeling of nanostructures.

Studying the static and dynamical behavior of elastic nanorod, nanobeam and nanoplates subject of many papers (Ansari et al., 2010; Akgöz and Civalek, 2013; Aydogdu and Filiz, 2011; Wang et al., 2006). One of the first applications of the nonlocal continuum theory in nanotechnology is the work presented by Peddieson et al. (2003). They used the nonlocal elasticity theory to develop

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nonlocal Euler–Bernoulli beam for different boundary conditions. Also, they are considering application of cantilever beam as microelectromechanical actuator. Lately, nonlocal theories for the Euler–Bernoulli, Timoshenko, Reddy and Levinson beams are derived by Reddy (Reddy, 2007) in a unique way using Hamilton's principle and nonlocal constitutive relation of Eringen. The author is obtain analytical solution of banding, vibration and buckling and showed the effect of nonlocal parameter on deflections, buckling load and natural frequencies. In the paper, presented by Reddy and Pang (Reddy and Pang, 2008), the equations of motion of Euler–Bernoulli and Timoshenko beam theories are reformulated by Eringen nonlocal theory, and then used to evaluate static bending, vibrations, and buckling response of carbon nanotubes with several boundary conditions. The influences of nonlocal parameter and aspect ratio on the natural frequency, static deflection and buckling load are considered. The small scale effect on the axial vibration of a tapered nanorod based on the nonlocal elasticity theory is studied by Danesh et al. (2012). The governing equations are solved by using the differential quadrature method for three type of boundary conditions, clamped–clamped (C–C), clamped–free (C–F) and fixed-attached spring boundary conditions. Also, it is show that the nonlocal effect plays an important role in the axial vibration of nanorods. The free vibration of double-nanorod system is investigated by Murmu and Adhikari (2010). Based on Eringen's nonlocal elasticity theory and methods of separations of variables, they obtained analytical solutions for natural frequencies for two types of boundary conditions, Clamped–-Clamped and Clamped–Free. A carbon nanotube embedded in an elastic medium was modeled by Aydogdu (2012) as a nanorod surrounded with elastic layers by using the Eringen's nonlocal elasticity theory. The author compared the longitudinal frequencies for the nonlocal and classical continuum models. Narendar and Gopalakrishnan (2011) considered the nonlocal effects in the axial wave propagation within the system of two nanorods coupled with an elastic layer. The authors studied the influence of small-scale (nonlocal) parameter and stiffness of the layer on axial wave propagation. Hsu et al. (2011) investigated the longitudinal frequencies of cracked nanobeams for different boundary conditions and using the theory of nonlocal elasticity. A wide study of the longitudinal, transversal and torsional vibration and instability was conducted by Kiani (2013) for a system of SWCNTs. Şimşek (2012) used a Galerkin approach to obtain the natural frequencies for the longitudinal vibration of axially functionally graded tapered nanorods. The author performed the analysis for nanorods with a variable cross-section, differently tapered ratios, material properties and boundary conditions. Longitudinal vibration of nanorods, which takes the nonlocal long-range interactions into account, was examined by Huang (2012). Chang (2012) considered the small-scale effects to investigate the axial vibration of elastic nanorods. The author used the differential quadrature method to solve the model equations. Filiz and Aydogdu (2010) analyzed the longitudinal vibration of carbon nanotubes with heterojunctions using the nonlocal elasticity for different lengths, diameters and chirality of heterojunctions. Karličić et al. (2015) performed a detailed analysis of the free longitudinal vibrational response of the system with two coupled viscoelastic nanorods and investigated the influence of different physical parameters on complex natural frequencies. Recently, Adhikari et al. (2013) examined the free and forced longitudinal vibration of the nonlocal nanorod by using two types of nonlocal damping models. The authors obtained the partial differential equation of motion in terms of axial displacements and then solved by analytical and finite element method. Exact analytical solutions for cut-off frequency are also obtained when the number of mode in the complex natural frequency tends to the infinity.

Damping properties appear in all nanostructures systems and help to better define suppression vibration behavior. Understanding their source is an important issue, not only for design and applications in nanoengineering practice but also to understand the inner workings of the nanomaterial's and nanostructures elements. Therefore, different technologies have been developed to investigated the damping effects on the vibration characteristics of damped or viscoelastic nanostructures (Imboden and Mohanty, 2014). Viscoelastic materials displaying both solid-like and fluid like characteristics, are common in polymeric structures. Energy dissipation or portion of energy storage from fluid-like part is irrecoverable and can be separated from energy of deformation using a complex modulus, which is represented by real and imaginary parts named storage and loss modulus, respectively. Thus, should be paid a more attention to the study of the dynamic behavior of the nanostructures with viscoelastic properties. The application of the nonlocal continuum theory to describe the internal and external damping effects in the structure elements at the nanoscale level have started recently. Lei et al. (2013a) proposed two type nonlocal dumped viscoelastic model of nanobeam based on nonlocal viscoelastic constitutive relations for vibration analysis. A transfer function methods is applied to obtain analytical solutions of free vibration for Euler–Bernoulli nanobeam with different boundary conditions. Also, the influences of material and geometric parameters on the complex eigenvalue are investigated. In the paper by Lei et al. (2013b) the dynamical behavior of nonlocal viscoelastic damped nanobeam has been investigated by using the Kelvin–Voigt viscoelastic model, velocity-dependent external damping and Timoshenko beam theory. The authors showed that nonlocal damped beams have maximum frequencies, called asymptotic frequencies, and also possess an asymptotic critical damping factor. The numerical results are presented on carbon nanotube example. In the paper by Paola et al. (2013) the dynamics of a nonlocal Timoshenko beam is presented. Nonlocal effects are modeled as long-range volume forces and moments mutually exerted by non-adjacent beam segments, that contribute to the equilibrium of any beam segment along with the classical local stress resultants. Also, model is provided with elastic and viscous long-range volume forces and moments which are linearly dependent on the product of the volumes of the interacting beam segments and on generalized measures of their relative motion, based on the pure deformation modes of the beam. The numerical results are presented for different values of nonlocal parameters. Vibration behavior of boron nitride nanotubes coupled by visco-Pasternak layer under a moving nanoparticle was proposed by Ghorbanpour Arani and Roudbari (2013) who investigated the nonlocal piezoelastic surface effect. Pouresmaeeli et al. (2013) reported on vibration characteristics of simply supported viscoelastic orthotropic nanoplates resting on viscoelastic foundation. The authors are obtained closed form solutions of complex frequencies which includes influence of nonlocal parameter and structural damping of the nanoplate and foundation. They showed that the frequency significantly decreases with increasing the structural damping.

By browsing the literature, the authors have found that some interesting papers about physics of multiple system of nanorods (Lao et al., 2002; Wen et al., 2003; Schulz et al., 2005). Nanorods growing from nanowire core can be viewed as multi-nanorod system Fig. 1. Mechanical modeling of those systems can be of great progress for their application and comprehension since experiments on nano-scale level cannot be well controlled. Therefore, this paper represents an extension of work Karličić et al. (2015), for systems of multiple coupled nanorods with viscoelastic properties. In the following of this work, it is presented an analytical solution of axial vibrations of a viscoelastic multi-nanorod system embedded