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Interlaminar stresses in piezoelectric laminated composite shells under electric, thermal and mechanical loads

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ABSTRACT

An analytical method is presented to investigate the influences of electric, thermal and mechanical loads on interlaminar stresses in piezoelectric laminated composite shells with two simply supported ends. Based on the geometrical and loading forms of piezoelectric composite laminated shells, the axisymmetrical model of a three dimensional orthotropic thermo-piezoelectric elastic problem is found to obtain an analytical solution containing some undetermined constants for a separate piezoelectric layer or fiber reinforced layer, where the reinforced direction of fiber layer and the stacking sequence of piezoelectric laminated composite shells may be arbitrary. The undetermined constants involved in the analytical solution are obtained by means of the continuity conditions between layers, the boundary conditions at internal and external surface of piezoelectric laminated composite shells and the supported conditions at two ends. Therefore, an exact solution for interlaminar stresses in piezoelectric laminated composite shells is obtained. The results show that the amplitude of interlaminar shear stresses in piezoelectric composite laminated shells can be reduced by choosing a particular values of electric filed for different stacking sequence, and the interlaminar shear stresses in piezoelectric laminated shells with fiber sub-layer $[\theta/-\theta]$ are also reduced to a smaller value by optimizing the reinforced direction of fiber sub-layer $[\theta/-\theta]$, so that it is easily to control the delamination failure of the piezoelectric fiber reinforced laminated shells.

1. Introduction

The physical behaviors of direct and converse piezoelectric effects of piezoelectric materials are often used as distributed sensors and actuators (Khdeir and Aldraihem, 2011; Ray and Reddy, 2005; Reddy, 1997; Sheng and Wang, 2010; Ray and Reddy, 2013). Therefore, composite laminated structures with piezoelectric layers are widely used in aerospace, automobile and other engineering areas, and are often subjected to combinations of electric, thermal and mechanical loads (Lam et al., 1997; Vel and Baillargeon, 2005; Dong and Wang, 2006; Ray and Mallik, 2004, 2005). To understand the distribution characteristic of interlaminar stresses in piezoelectric laminated composite structures is of importance for predicting and preventing the delaminated failure of the piezoelectric laminated composite structures due to the interlaminar stress in composite laminated structures is a main cause to induce the delamination damage of the composite laminated structures (Zhu et al., 2008; Yang et al., 2014; Chen et al., 2004).

Many studies on the distribution characteristic of interlaminar stresses in composite laminated structures without piezoelectric player, under various loads, have been reported in Refs. (Hamidreza and Mohammadreza, 2011; Plagianakosa and Saravanosb, 2009; Heung Soo et al., 2008; Asghar and Arash, 2007; Roosa and Kressa, 2007; Davood Mousanezhad et al., 2013; Ahn and Kim, 2013) so far. An analytical solution is presented to describe the interlaminar stresses characteristics near the free edges of generally laminated composite plates under the extension and bending loads, based on the reduced elasticity displacement field for a long laminate (Hamidreza and Mohammadreza, 2011). Based on a higher-order layerwise theoretical framework, the interlaminar shear stresses in thick composite and sandwich composite plates are predicted (Plagianakosa and Saravanosb, 2009). In Ref.(Heung Soo et al., 2008), it is seen that a stress function-based variational method is developed to investigate the interlaminar stresses near the dropped plies, based

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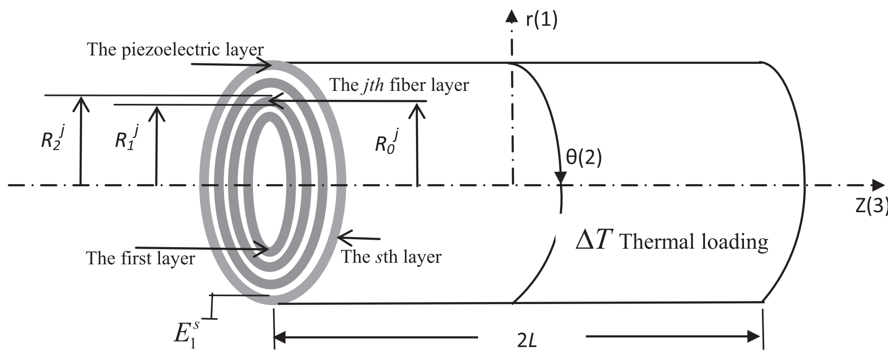

Fig. 1. The calculation model for interlamilar stresses in piezoelectric laminated shells under radial electric field, pressure loading and thermal loading.

on expanding stress functions in terms of harmonic series to the out-of-plane direction. An analytical solution is presented in Ref.(Asghar and Arash, 2007) to describe the edge-effect problem of long antisymmetric angle-ply composite laminates subjected to extensional and/or torsional loads, using the first-order shear deformation theory of plates and Reddy's layerwise theory. An effective model for interlaminar normal stress distribution in singly curved and moderately thick laminates is presented to model the influence of interlaminar shear stresses (Roosa and Kressa, 2007), and this new model and a FEM model are used to predict the critical delamination loads observed in experiments which are also described. Reference (Davood Mousanezhad et al., 2013) gives an analytical method to solve the interlaminar stresses in long symmetric laminated composite plates subjected to shearing loads, based on an improved first-order shear deformation theory (IFSDT) and a simplified IFSDT (SIFSDT). In Ref. (Ahn and Kim, 2013), the p-convergent global–local model based on layerwise theory is presented to solve the interlaminar stresses around a circular hole of composite laminates under tension.

Recently, because piezoelectric composite laminated structures are widely utilized in automobile, aerospace and other engineering areas, the distribution characteristics of interlaminar stresses in the piezoelectric composite laminated structures are followed with much interest (Izadi and Tahani, 2010; Huang and Kim, 2014; Brischetto and Carrera, 2012; Wang et al., 2005). Reference (Izadi and Tahani, 2010) presented a solving method to analyze the interlaminar stresses of piezo-bonded symmetric laminates under electric fields, based on a stress function satisfying the traction free boundary conditions and surface free conditions. An analytical method is presented to solve the interlaminar stresses of general cross-ply laminates with piezoelectric layers as actuators under transverse mechanical loads, where the singular behavior of interlaminar normal and shear stresses in the boundary region near the edges of the laminate are described (Huang and Kim, 2014). Coupled thermo-electro-mechanical analysis of smart plates embedding composite and piezoelectric layers is given in Ref. (Brischetto and Carrera, 2012). Reference (Wang et al., 2005) described hygrothermal effect on dynamic interlaminar stresses in laminated plates with piezoelectric actuators.

However, because of the complexity to solve interlaminar stresses in piezoelectric laminated composite shells under coupled thermal, electric and mechanical loads, analytical solutions for interlaminar stresses in piezoelectric laminated composite shells are few in the literature so far. In this paper, an analytical method is presented to solve the problem of interlaminar stresses in piezoelectric laminated composite shells with $[\pm\theta]$ fiber reinforced sub-layer, under coupled thermal, electric and mechanical loads. The results show the effect of temperature, applied electric fields, radical pressure and stacking sequence on the interlaminar stresses in piezoelectric laminated composite shells. The analytical solution in this paper may be used as a benchmark for approximate solutions and an optimizing design reference for predicting and preventing the delaminated failure of piezoelectric laminated composite structures.

2. Governing equations and solving process

The geometry and coordinate system of thermo-piezoelectric composite laminated cylindrical shells are shown in Fig. 1, where a subscript j is used to describe each layer (piezoelectric layer or fiber reinforced layer) and L is the half-length of the thermo-piezoelectric composite laminated cylindrical shells. For thermo-piezoelectric composite laminated cylindrical shells containing s layers, as shown in Fig. 1, h_j, R_1^j, R_2^j and R_o^j express the thickness, internal radius, external radius, and middle radius of the jth layer ($j = 1, 2, 3, \ldots s$), respectively.

Considering the axisymmetrical behaviors of geometrical shape and loading forms, the physical equations of the jth orthotropic piezoelectric or fiber reinforced cylindrical sub-shell under thermal, electrical and radial pressure loads are written as (Lekhniskii, 1981)

$$\begin{Bmatrix} \sigma_r^j \\ \sigma_\theta^j \\ \sigma_z^j \\ \tau_{rz}^j \end{Bmatrix} = C_{11}^j \begin{bmatrix} 1 & C_1^j & C_2^j & 0 \\ C_1^j & C_3^j & C_4^j & 0 \\ C_2^j & C_4^j & C_5^j & 0 \\ 0 & 0 & 0 & C_6^j \end{bmatrix} \begin{Bmatrix} \varepsilon_r^j - \alpha_r^j \Delta T^j \\ \varepsilon_\theta^j - \alpha_\theta^j \Delta T^j \\ \varepsilon_z^j - \alpha_z^j \Delta T^j \\ \gamma_{rz}^j \end{Bmatrix} - \begin{bmatrix} e_{11}^j & 0 & 0 \\ e_{21}^j & 0 & 0 \\ e_{31}^j & 0 & 0 \\ 0 & e_{42}^j & 0 \end{bmatrix} \begin{Bmatrix} E_1^j \\ E_2^j \\ E_3^j \end{Bmatrix} \tag{1}$$