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Nonlinear thermal dynamic buckling of FGM beams

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ABSTRACT

Based on the nonlinear dynamic analysis, dynamic buckling and imperfection sensitivity of the FGM Timoshenko beam subjected to sudden uniform temperature rise are studied. Initial geometric imperfection of the beam is also taken into account. It is assumed that during deformation the beam is resting over a conventional three-parameter elastic foundation with softening/hardening cubic non-linearity. The analysis is performed with considering temperature dependency assumption of each thermo-mechanical property of the FGM beam. The governing nonlinear dynamic equations are derived based on the generalized Hamilton principle. In the spatial approximation of the problem, a set of ordinary differential equations in time is obtained by the conventional multi-term Ritz method. These equations are converted into a set of algebraic equations by utilizing the Newmark family of time approximation scheme. The obtained non-linear algebraic equations are solved via the well known Newton–Raphson iterative scheme. The Budiansky–Roth criterion is used to detect the unbounded motion type of dynamic buckling. Results reveal that for beams with stable post-buckling equilibrium path, no dynamic buckling occurs according to the Budiansky–Roth criterion. However, dynamic buckling may occur for the FGM beams resting on sufficiently stiff softening elastic foundation due to their unstable post-buckling equilibrium paths.

1. Introduction

Beam like structures are known as a fundamental part of more complex structures. Vast application of beams is observed in various aspects of civil, mechanical, and structural engineering. Therefore, understanding the behavior of beams under various loading conditions is an essential tool to establish a reliable design. Stability analysis of beams under static or dynamic loads always stands as a branch of such essential tools.

By development of functionally graded materials (FGMs) as a class of novel materials, recent investigations of researches are focused on the beams made of FGMs. FGM structures are born mainly to withstand the high thermal environment and temperature gradients. Compatible with such demand, more investigations are devoted to thermal stability analysis of FGM beams instead of mechanical stability analysis.

Static stability of FGM beams under thermal loading has been studied through the past decade by various investigators. The term *static stability*, here, is used for both linear stability analysis which results in bifurcation detection and non-linear stability which yields the load–deflection equilibrium path. In linear buckling analysis of FGM beams, Kiani and Eslami (2010, 2013) evaluated the thermal bifurcation loads of slender and moderately thick through-the-thickness FGM beams. Eigenvalue solution of linearized stability equations is solved to detect the non-trivial states associated to bifurcation type of buckling. In these researches it is shown that, due to the asymmetrical distribution of material properties in conventional FGMs, bifurcation phenomenon occurs under special cases. In two other studies, Kiani et al. (2011a,b) enriched the FGM beam with two piezoelectric actuator layers to enhance the buckling temperatures of the beam. Numerical results of this study reveal the inability of piezoelectric layers to control the bifurcation temperatures due to the high stiffness of the FGM beam and thicker configuration of FGM layer in comparison to piezoelectric layers. Employing a higher order beam theory which captures the through-the-thickness normal and shear strains, Wattanasakulpong et al. (2011) implemented the Ritz formulation for thermal buckling of FGM beams. In this work, however, conditions for an FGM beam to obey the bifurcation type of buckling is not examined. Bhangale and Ganesan (2006) examined the thermal bifurcation of sandwich FGM beams with viscoelastic core based on a finite elements formulation.

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Non-linear stability behavior of FGM beams under in-plane thermal loading is also studied by implementation of various numerical methods. Zhao et al. (2007) reformulated the non-linear governing equilibrium equations of an FGM beam to a new system of ordinary differential equations suitable for shooting method. Numerical results of this study reveal the unique and stable equilibrium path for simply supported FGM beams under uniform thermal loading. Li et al. (2006) also applied the shooting method to the non-linear equilibrium equations of a Timoshenko beam corresponding to the post-buckling equilibrium path of the beam. Ma and Lee (2012) and Kargani et al. (2013) decoupled the non-linear equilibrium equations of the FGM beam and presented a closed form expression to trace the equilibrium paths of the beam under in-plane thermal loading. Their numerical results, however, are valid within the intermediate post-buckling range in which buckled shape governs the post-buckled shape. Based on a single term Ritz method, Anandrao et al. (2010) traced the equilibrium path of an FGM beam under in-plane thermal loading. The case of an FGM beam under uniform temperature rise loading is considered in this research. An approximate closed form solution is given which gives the midspan deflection as a function of temperature rise parameter. Different behaviors are observed for simply supported and clamped edges due to the stretching–bending coupling stiffness. Suitable only for FGM beams with both edges clamped, a single term Galerkin method is also developed by Fu et al. (2012). Instead of adopting the flexural beam theories, Levyakov (2013) analyzed the non-linear response of FGM beams using an elastica solution. It is shown that at higher temperature levels in-plane force through the beam may be tensile, which results in snapping phenomenon. To eliminate the stretching–bending coupling through the formulation, Zhang (2014) used a neutral surface based formulation applied to the third order shear deformable beam theory formulation. Esfahani et al. (2013, 2014) discretized the non-linear equilibrium and motion equations of an FGM beam employing the generalized differential quadrature method. Thermal post-buckling and vibration of thermally post-buckled FGM beams are analyzed in these researches. In these studies, influence of non-linear elastic foundation and temperature dependency of the constituents are also taken into account. Ma and Lee (2011) also examined the thermal stability and vibration of thermally post-buckled behavior of FGM beams suitable for moderately thick beams. Present authors analyzed the static and dynamic limit load type of buckling in an FGM beam resting on non-linear softening elastic foundation (Ghiasian et al., 2013). Only the case of a beam with both edges clamped is investigated. It is shown that a sufficiently soft elastic medium results in unstable post-buckling equilibrium branch and forces the beam to follow the violate type of buckling.

In comparison to static buckling, investigations on the dynamic buckling of FGM structures are less reported through the open literature. Dynamic buckling is a complicated behavior which should be explored through the response of non-linear motion equations of the structure. Definition of a dynamically buckled structure strongly depends on the criterion. A wealth review on the concept of dynamic buckling and its applications to solid structures is reported in a review paper by Simitses (1987) and also documented in a book by Simitses (1990). Among the most well-known and suitable criteria, the motion equation criterion of Budiansky and Roth (1510) (which is also known as the Budiansky–Hutchinson for initially imperfect structures Hutchinson and Budiansky, 1966), the phase–plane approach of Hoff–Hsu (Hsu, 1967), the modified total potential energy approach of Hoff–Simitses (Simitses, 1967), displacement control approach of Volmir (1972), quasi-bifurcation dynamic buckling of Kleiber et al. (1987) and the criterion of Kubiak (2007) or Kounadis et al. (1997) are the most frequently used ones. Each criterion has its own advantages and shortcomings. Meanwhile, the Budiansky–Roth criterion is the most popular one since is easy to be used in computer programming and has no limitation in structural analysis (Simitses, 1987).

Referring to the thermal dynamic buckling, Budiansky–Roth criterion is applied successfully to cylindrical shells (Shariyat, 2008a,b; Mirzavand et al., 2010; Mirzavand et al., 2013; Shariyat and Elsami, 2000, 2002), plates (Shuka and Nath, 2002; Shariyat, 2009) and spherical caps (Prakash et al., 2007; Sundararajan and Ganapathi, 2008). However, to the best of authors knowledge, no work has been reported yet on thermal dynamic buckling of beams made of FGMs or even homogeneous ones based on the Budiansky–Roth criterion. The only available research on thermal dynamic buckling of FGM beams under rapid heating is reported recently by the present authors (Ghiasian et al., 2013). This research, however, is developed based on the Hoff–Simitses criterion. As known, the Hoff–Simitses criterion yields only the magnitude of critical temperature in which dynamic buckling phenomenon occurs and does not establish the dynamic sense of the structure prior or at the onset of buckling. The present research examines the thermal dynamic buckling and imperfection sensitivity of FGM beams subjected to uniform rapid heating. Temperature dependency, initial imperfections and contact of a three-parameter conventional non-linear elastic foundation are also taken into account. Timoshenko beam theory, geometrical non-linearity in the von-Kármán sense and uncoupled thermoelasticity constitutive law of a continuum medium are incorporated together to establish the Hamiltonian of the system. The conventional multi-term Ritz method is applied to the Hamiltonian of the system to establish the matrix representation of the non-linear motion equations. To solve the highly coupled non-linear equation in time domain, a hybrid Newmark–Newton–Raphson method is applied to the associate equations which traces the temporal evolution of beam deformations. Solution method is general and may be used for arbitrary grading profile and edge supports. Budiansky–Roth criterion is applied successively to the motion equations and detect the buckling state. It is shown that FGM beams do not undergo any type of thermal dynamic buckling in the Budiansky–Roth sense. However, a sufficiently soft nonlinear elastic foundation violates the response of the beam and results in the unbounded motion type of dynamic buckling.

2. Fundamental equations of the FGM beam

A beam with length L and thickness h is considered in the conventional Cartesian coordinate system (x, z), as shown in Fig. 1. In this system x and z represent, respectively, the longitudinal and transversal directions.

2.1. Description of material properties

Beam is made of FGMs and its material properties are assumed to be graded through the thickness. According to a simple power

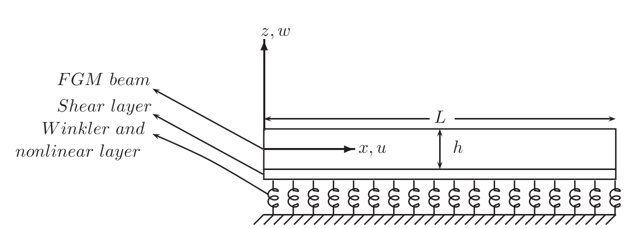

Fig. 1. Coordinate system and geometry of an FGM beam resting on elastic foundation.