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Static and free vibration analyses of functionally graded sandwich plates using state space differential quadrature method

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ABSTRACT

This study presents static and free vibration behaviors of two type of sandwich plates based on the three dimensional theory of elasticity. The core layer of one type is functionally graded material (FGM) with the isotropic face sheets whereas in second type, the core layer is isotropic with the face sheets FGM. The effective material properties of FGM layers are estimated to vary continuously through the thickness direction according to a power-law distribution of the volume fractions of the constituents. By using differential equilibrium equations and/or equations of motion as well as constitutive relations, state-space differential equation can be derived. In the case of simply supported condition, applying Fourier series to the quantities along the in-plane coordinates, governing equation can be solved analytically and for the other edges condition, a semi analytical solution can be obtained by using differential quadrature method (DQM) along the in-plane coordinate as well as state spaces technique in the thickness direction. Accuracy and exactness of the present approach is validated by comparing the numerical results with the results of published literature. Moreover, the influences of volume fraction, width-to-thickness ratios and aspect ratio on the vibration and static behaviors of plate are investigated.

1. Introduction

Sandwich plates are widely used in modern engineering applications, especially in mechanical industries, nuclear reactors and aeronautical industry due to its lightweight, high stiffness, high structural efficiency and strength. Analytical and experimental studies on sandwich structures show that delamination occurs at the interface of sheets and the core, In addition, face sheet failure and shear cracking of the core occurs, particularly when subjected to transverse loading. This is because their material properties are mismatching in the layers interface. It is possible to overcome this problem by using FGM sandwich plate. FGM sandwich plates can exist in two forms: one form has functionally graded faces with homogeneous core and in the other form, homogeneous faces and functionally graded core is used. Recently, many researchers have studied mechanical properties and behavior of FG sandwich structures. Based on three-dimensional elasticity equations, Liu (2000) used the DQM to study the static behavior of thick rectangular laminated composite plates. Reddy and Cheng (2001) presented three-dimensional thermo-mechanical deformations of simply supported functionally graded rectangular plates by using an asymptotic method. Anderson (2003) carried out an analytical three-dimensional elasticity solution for a sandwich panel consists of orthotropic face sheets bonded to an isotropic FGM core subjected to transverse loading. Ferreira et al. (2005) analyzed static deformation of a simply supported functionally graded plate by using third-order shear deformation theory and a meshless method. Li et al. (2008) used the three-dimensional theory of elasticity to investigate free vibration of simply supported and clamped edges sandwich plates with volume fraction distribution of constituents according to simple power law. Kant et al. (2008) used two-point boundary value problem (BVP) governed by a set of linear first-order ordinary differential equations (ODEs) through the thickness of a laminate to present semi-analytical model for estimation of stresses and displacements in composite and sandwich laminates. Based on classical and mixed advanced models, static analysis of sandwich plates with material properties according to the Legendre polynomials was carried out by Brischetto (2009) and Tu et al. (2010) presented bending and vibration analysis of laminated and sandwich composite plates by using finite element method. They used the higher order theory to account for rotary inertia effects and

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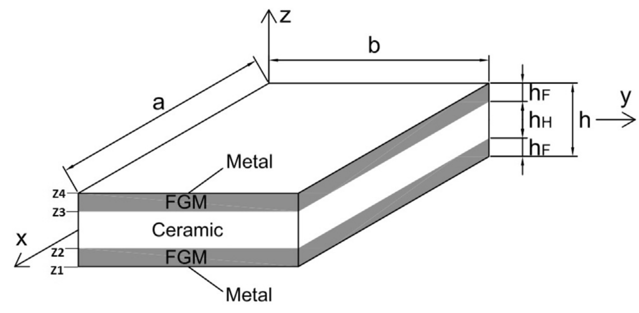
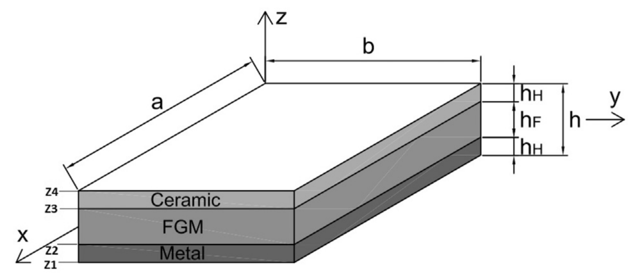
Nomenclature

a, b, h	plate dimensions in x-, y- and z-directions, respectively
E_0, ρ_0	Young modulus and material density of metal layer, respectively
$C_{ij} = C_{ij}(z)$ (i, j = 1,2,...,6)	material elastic constants
$E(z)$, E_c, E_m	Young's modulus of FGM, ceramic and metal layers, respectively
m, n	half wave number along longitudinal and width direction, respectively
P	volume fraction index
u, v, w	displacement components in the x-, y- and z-direction, respectively
V_i (i= 1, 2, 3)	volume fraction
σ_i (i = x, y, z)	normal stresses
τ_{xy}, τ_{xz}, τ_{yz}	shear stresses
ε_i (i = x, y, z)	normal strains
γ_{xy}, γ_{xz}, γ_{yz}	shear strains
λ, μ	lame constants
$\rho(z)$, ρ_c, ρ_m	material density of FGM, ceramic and metal layers, respectively

parabolic distribution of the transverse shear strains through the thickness of plate. Nguyen-Xuan et al. (2011) presented static, free vibration and buckling behavior of FGM plate by using an edge-based smoothed finite element method. Abdelaziz et al. (2011) used the high-order shear deformation theory (HSDT) to analyze static behavior of FG sandwich plate. Natarajan and Manickam (2012) studied bending and free vibration of sandwich FGM plate by using QUAD-8 shear flexible element developed based on higher order structural theory. Neves et al. (2012) presented static analysis of FG sandwich plates by using hyperbolic sine term for the in-plane displacements and a quadratic function of thickness coordinate for transverse displacement. Khalili and Mohammadi (2012) used Hamilton's principle to investigate free vibration of sandwich plates with FG face sheets for various thermal environments. They assumed that the material properties of FG face sheets are temperature-dependent by a third-order function of temperature. Based on two-dimensional shear deformation theory, Xiang et al. (2013) analyzed free vibration of sandwich plate with FG face sheets and homogeneous core by employing meshless global collocation method. Loja et al. (2013) studied static and free vibration behavior of FG sandwich plate using B-spline finite strip element models based on various shear deformation theories and assumption of material properties according to Mori–Tanaka formulation. Castellazzi et al. (2013) investigated static behavior of FG plates by using a nodal integrated finite element approach based on the first-order shear deformation theory. Neves et al. (2013) employed a higher-order shear deformation theory and meshless technique to analysis static, free vibration and buckling behaviors of isotropic and sandwich FG plates with simply supported boundary conditions. Based on a refined trigonometric shear deformation theory, Tounsi et al. (2013) investigated thermoelastic bending analysis of simply support FG sandwich plates. Based on theory of elasticity, Alibeigloo (2014) used Fourier series expansions and state-space technique to investigate bending behavior of a simply supported sandwich panel with FG core subjected to thermo-mechanical load. Jin et al. (2014) presented a three-dimensional exact solution for free vibrations of arbitrarily thick FG rectangular plates with general boundary conditions.



a. Type A



b. Type B

Fig. 1. Geometry and coordinate system of sandwich plate.

According to the comprehensive literature survey, there is no work dealing with the three dimensional static and free vibration characteristics of functionally graded sandwich plates with various edges boundary condition. In this paper we present three dimensional static and free vibration analysis of simply supported FGM sandwich rectangular plate analytically by using Fourier series solution. Moreover, by using DQM along the in-plane coordinates and state space technique along the thickness direction, analysis is carried out semi analytically.

2. Problem formulation

In this investigation two types of rectangular FG sandwich plates with uniform thickness h, length a, and width b (Fig. 1) and following lay-up are considered.

- Type A (Fig. 1a): Isotropic core layer and FGM face sheets with the following volume fraction of constituents:

$$V_1 = \left(\frac{z - z_1}{z_2 - z_1}\right)^P, \quad z \in [z_1, z_2]$$
$$V_2 = 1, \qquad z \in [z_2, z_3] \tag{1}$$
$$V_3 = \left(\frac{z - z_4}{z_3 - z_4}\right)^P, \quad z \in [z_3, z_4]$$