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Stress-based criteria for brittle fracture in key-hole notches under mixed mode loading

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ABSTRACT

Two stress-based failure criteria were developed in the present research to predict mixed mode I/II brittle fracture in engineering components weakened by a key-hole notch. The first criterion was based on the well-established maximum tangential stress (MTS) concept, successfully proposed and utilized several times in recent years for different notch features. The second one was on the basis of the mean-stress (MS) concept which has been frequently used for predicting pure mode I fracture in notched domains. The results of the criteria were represented in the form of fracture curves and the curves of fracture initiation angle in terms of the notch stress intensity factors. To verify the validity of the fracture models, the theoretical predictions were compared with a large bulk of experimental data, reported in literature, on the fracture of rectangular isostatic graphite plates weakened by central key-hole notches of five different tip radii. It was found that while the total accuracies of the criteria are very good, for small notch tip radii, the MTS criterion provides generally better mixed mode notch fracture toughness results than the MS criterion. Conversely, the MS criterion works much better than the MTS model for larger notch tip radii. Dealing with fracture initiation angle, both the criteria could estimate the experimental results successfully.

1. Introduction

Notches are famous man-made stress raisers which are inevitably introduced in engineering components and structures for different purposes such as to join two or more structural elements etc. Due to the stress concentration, cracks and other types of failure evidences may be detected in engineering components at the notch border. During periodic inspections and major overhauls, cracks and damages, detected normally by visual inspections and non-destructive testing (NDT) methods, must be removed from the notched components by different methods such as welding etc. For small cracks and damages nucleated from the notch edge, a common repair method is to remove the cracks by means of drilling a hole having the radius of normally equal to the crack length. Dealing with extensively-used U-shaped notches, this repair method can change the original notch shape to a key-hole notch or a nipple-shaped notch depending on the ratio *U-notch tip radius/crack length*. Nipple-shaped and key-hole notches are achieved for the ratios larger and smaller than one, respectively. Since the resulted notch feature is different from the original one (i.e. U-shaped notch), the stress distribution around the new notch would be less or more different from the U-notch needing a new failure assessment for resuming the component to be reliably in service.

Brittle fracture is a catastrophic failure mode that occurs often in brittle and quasi-brittle materials. Such materials like graphite materials, ceramics, ceramic-based composites etc. are usually utilized in engineering structures as heat shields. Moreover, many high-strength metallic materials such as some grades of commercial steels are utilized in various engineering applications for bearing great loads. These materials exhibit normally quasi-brittle behavior with moderately low ductility and they are prone to sudden fracture.

Dealing with brittle fracture in notched components, particularly V- and U-notched ones, most of the investigations have been performed by means of the notch fracture mechanics (NFM) which is, in fact, an extension of the classical fracture mechanics of cracked domains to notched ones. Under pure mode I loading conditions, the fracture behavior of sharp and rounded-tip notches has been studied in the past by several researchers both theoretically and experimentally (see for instance Gogotsi, 2003; Knesl, 1991; Nui et al., 1994; Seweryn, 1994; Strandberg, 2002). Generally, either being stress-based, energy based or even a combined stress-energy based, five main failure theories exist in open literature for assessing

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fracture in notched members, namely the cohesive zone model (CZM) (Gomez and Elices, 2003; Gomez et al., 2000), the strain energy density (SED) (Lazzarin and Zambardi, 2001; Lazzarin et al., 2003, 2009a; Yosibash et al., 2004; Berto et al., 2011), the generalized J-integral (Livieri, 2003, 2008; Matvienko and Morozov, 2004; Berto and Lazzarin, 2007; Barati et al., 2009; Barati and Alizadeh, 2011; Becker et al., 2012), the finite fracture mechanics (FFM) (Carpinteri et al., 2008), the point-stress (PS) and the mean-stress (MS) criteria (Ayatollahi and Torabi, 2010a,b; Torabi, 2012, 2013a,b,c; Torabi et al., 2013). Among the above-mentioned theories, the CZM, FFM, PS and MS models present their predictions simply in the form of $K_{\mathrm{I}}^{*} = K_{\mathrm{Ic}}^{*}$; an Irwin-like model $K_{\mathrm{I}} = K_{\mathrm{Ic}}$ for sharp crack problems. K_{I}, K_{Ic}, K_{I}^{*} and K_{Ic}^{*} are the stress intensity factor (SIF), the plane-strain fracture toughness of material, the notch stress intensity factor (NSIF) and the notch fracture toughness (NFT), respectively. A closed-form expression like that mentioned above has been suggested by Gomez and Elices (2003) for predicting mode I NFT of sharp V-notches by means of CZM. They verified the validity of their model by using mode I fracture test results obtained from V-notched samples of PMMA. Similar approach has been utilized by Gomez et al. (2000) to assess mode I brittle fracture in U-notched PMMA specimens. Carpinteri et al. (2008) have also proposed an expression based on the FFM failure concept to predict well the load-carrying capacity of V-notched rectangular beams made of PMMA and subjected to three-point bending. Most of the recent works in which the mode I fracture of notched components has been estimated by closed-form NFTs, have dealt with the PS and the MS criteria. Ayatollahi and Torabi (2010a) developed two Irwin-like expressions based on the PS and MS models by which the fracture toughness of several V-notched PMMA and ceramic samples was successfully predicted. Furthermore, they utilized well the MS criterion for predicting mode I fracture onset in wide range of polycrystalline graphite specimens weakened by rounded-tip V-notches (Ayatollahi and Torabi, 2010b). Torabi (2013a) reformulated the MS and PS criteria for U-notches and found two closed-form expressions for the mode I NFT. He verified his expressions by the experimental results reported in literature by Berto et al. (2012) on mode I fracture of U-notched rectangular plates made of isostatic poly-granular graphite. More recently, the same expressions have been successfully utilized by Torabi et al. (2013) to predict the experimental NFT values for the U-notched Brazilian disk (UNBD) specimens made of polycrystalline graphite. Additionally, the expressions of MS and PS criteria have been modified for ductile materials by means of the novel equivalent material concept (EMC) (Torabi, 2012) and utilized to predict tensile load-carrying capacity of U-notched ductile steel plates subjected to three-point bending (Torabi, 2013b). As a more applied work, ultimate tensile loads have been successfully predicted for ductile steel bolts containing V-shaped threads by means of the simple expression of the MS–EMC criterion (Torabi, 2013c).

Under mixed mode loading conditions, however, the number of failure models is much limited. The most popular fracture criteria that exist in literature are mainly the generalized maximum tangential stress (GMTS) (Ayatollahi and Torabi, 2009, 2010c, 2011a,b; Ayatollahi et al., 2011; Torabi, 2013d) and the strain energy density (SED) (Yosibash et al., 2006; Chen and Ozaki, 2008; Priel et al., 2008; Tovo et al., 2006; Gomez et al., 2007, 2008, 2009a,b; Berto and Ayatollahi, 2010; Lazzarin et al., 2009b). These criteria have been frequently utilized to estimate well the load-carrying capacity of V- and U-notched specimens made of various brittle materials. The GMTS criterion, so-called in different papers as V-notched MTS (V-MTS) (Ayatollahi and Torabi, 2010c, 2011a,b; Ayatollahi et al., 2011), U-notched MTS (UMTS) (Ayatollahi and Torabi, 2009; Torabi, 2013d) etc., is, in fact, the extension of the classical MTS criterion proposed originally by Erdogan and Sih (1963) for predicting mixed mode brittle fracture in cracked bodies. The GMTS criterion has been frequently verified by means of extensive fracture test results obtained from notched specimens made of different brittle materials, such as PMMA (Ayatollahi and Torabi, 2009, 2010c) and (Ayatollahi et al., 2011), soda-lime glass (Ayatollahi and Torabi, 2011b), polycrystalline graphite (Ayatollahi and Torabi, 2011a; Torabi, 2013d) etc.

Dealing with brittle fracture in key-hole notches, however, two papers have been already published in open literature. Kullmer and Richard (2006) published a paper in which the fracture loads of compact-tension-shear-notch (CTSN) specimens made of PMMA have been theoretically estimated by means of a stress-based failure criterion. Lazzarin et al. (2013) carried out extensive fracture experiments on rectangular isostatic graphite plates containing central key-hole notches of various notch tip radii under pure mode I and mixed mode I/II loading. The experimentally obtained fracture loads were successfully predicted by using the well-known SED criterion (Lazzarin et al., 2013).

In this research, the well-known maximum tangential stress (MTS) failure concept was extended to key-hole notched domains and the mixed mode Key-MTS criterion was developed. Moreover, the well-established mean-stress (MS) failure criterion, suggested and utilized mainly for V- and U-notched components under pure mode I loading, was formulated for the first time for key-hole notches under mixed mode loading conditions (the Key-MS criterion). Both the criteria provide their results in form of the fracture curves and the curves of fracture initiation angle, in terms of the notch stress intensity factors capable of predicting the notch fracture toughness and the fracture initiation angle of key-hole notched members under mixed mode I/II loading. In order to verify the fracture criteria, the theoretical results were compared with a great bulk of experimental results, reported in literature (Lazzarin et al., 2013), on the fracture of rectangular graphite plates containing central key-hole notches of different tip radii. The comparison indicated that while the total accuracies of the fracture criteria are very good; for small notch tip radii, the Key-MTS criterion provides generally better notch fracture toughness results than the Key-MS criterion. Conversely, Key-MS works much better than Key-MTS for greater notch tip radii. Moreover, the two criteria could estimate successfully the experimental results of fracture initiation angle.

2. Brittle fracture criteria

In this section, the in-plane elastic stress distribution around a key-hole notch is considered to formulate the Key-MTS and the Key-MS fracture criteria. Kullmer (1992) was the first one who formulated the stress distribution around a key-hole notch. The elastic stress distribution around a V-notch with end hole has also been presented in a paper by Zappalorto and Lazzarin (2011) in which the formulas can be reduced to achieve the stress distribution for key-hole notches by considering zero value for the V-notch angle. They stated that their formulas coincide with those reported by Kullmer (1992). Since only the tangential component of the stress field is required for developing the fracture criteria, such component is taken from Kullmer (1992) and Zappalorto and Lazzarin (2011) as follows:

$$
\begin{aligned}
\sigma_{\theta\theta} = & \frac{K_{\mathrm{I}}^{\mathrm{key}}}{2\sqrt{2\pi r}}\left\{\left[\frac{3}{2}\cos\frac{\theta}{2}+\cos\frac{\theta}{2}\left(\frac{\rho}{r}\right)+\frac{3}{2}\cos\frac{\theta}{2}\left(\frac{\rho}{r}\right)^{2}\right]\right.\\
& \left.+\left[\frac{1}{2}\cos\frac{3\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\left(\frac{\rho}{r}\right)+\frac{5}{4}\cos\frac{3\theta}{2}\left(\frac{\rho}{r}\right)^{3}\right]\right\}\\
& +\frac{K_{\mathrm{II}}^{\mathrm{key}}}{2\sqrt{2\pi r}}\left\{\left[\frac{3}{2}\sin\frac{\theta}{2}+\sin\frac{\theta}{2}\left(\frac{\rho}{r}\right)+\frac{3}{2}\sin\frac{\theta}{2}\left(\frac{\rho}{r}\right)^{2}\right]\right.\\
& \left.+\left[\frac{3}{2}\sin\frac{3\theta}{2}+\frac{3}{4}\sin\frac{3\theta}{2}\left(\frac{\rho}{r}\right)+\frac{15}{4}\sin\frac{3\theta}{2}\left(\frac{\rho}{r}\right)^{3}\right]\right\}
\end{aligned}
\tag{1}
$$