European Journal of Mechanics A/Solids 49 (2015) 35–48

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A dispersive nonlocal model for shear wave propagation in laminated composites with periodic structures

H. Brito-Santana [a], Yue-Sheng Wang [a,*], R. Rodríguez-Ramos [b], J. Bravo-Castillero [b], R. Guinovart-Díaz [b], Volnei Tita [c]

[a] Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, China
[b] Faculty of Mathematics and Computing Sciences, University of Havana, San Lázaro esq. L, Vedado, Havana CP10400, Cuba
[c] Department of Aeronautical Engineering, Engineering School of São Carlos, University of São Paulo, Av. João Dagnone, 1100 São Carlos, SP, Brazil

ARTICLE INFO

Article history:
Received 23 November 2013
Accepted 26 May 2014
Available online 27 June 2014

Keywords:
Periodic composite material
Wave dispersion
Dynamic asymptotic homogenization

ABSTRACT

In this paper, the problem of shear-wave propagation with oblique incidence in a triclinic laminated composite with perfect contact between the layers and periodic distribution between them is studied. An asymptotic dispersive method for the description of the dynamic processes is proposed. By assuming a single-frequency dependency of the solution for the two-dimensional wave equation in a periodic composite material, the higher-order terms for the displacement in asymptotic expansions are studied. Analytic solution for the average model is presented with the graphical illustration for a boundary problem. Numerical examples show that the dispersion curve is in good agreement with the results in previous literatures. The effects of the unit cell size, wave number and incident angle on the wave propagation and dispersion relation are also examined.

1. Introduction

Many studies in the theory of composite materials are based on the exploitation of the classical continuum model implying that the original heterogeneous medium can be simulated by a homogeneous one with certain homogenized (so-called effective) properties. Such approach comes naturally from the hypothesis of the perfect rate of microheterogeneity of the composite structure when the microscopic size l of heterogeneities is supposed to be essentially smaller than the macroscopic size L of the whole sample so that in the first approximation one may assume $l/L = 0$. However, this limit is never reached for most practical problems, and in real composites the microstructural scale effects may result in specific nonlocal phenomena, which cannot be predicted in the frame of the homogenized medium theory. Simulating a composite material as a homogeneous medium and determining the effective elastic and inertial coefficients by homogenization techniques is one of the most important steps for the analysis and design of this material as well as for the nondestructive detection of defects in it. Several different methods have been developed based on high frequency homogenization (Craster et al., 2010; Nolde et al., 2011), on a micromechanical homogenization (Nemat-Nasser and Srivastava, 2011), and by using lattice model approximation (Carta and Brun, 2012).

The elastostatic response of composites has been understood to be non-local in space (Hill, 1965; Beran, 1968). However, in the context of inhomogeneous elastodynamics the effective constitutive relations are non-local in both space and time as investigated by Willis (2009, 2011, 2012). Field integration-based homogenization for calculating these overall dynamic properties of composites has been proposed by several researchers.

There are two categories of homogenization techniques for dynamic problems. The first category employs techniques based on the asymptotic expansion of the displacement field with respect to the representative volume of the composite (Mazur-Sniady et al., 2004; Smyshlyaev, 2009; Chen and Fish, 2001). In the second category, the techniques are based on the analysis of the multiple scattering caused by plane waves propagating into the composite (Kim, 2004; António et al., 2005; Fang et al., 2009; Wang et al., 2009; Molero et al., 2011). The present work concerns the first category of homogenization techniques.

The classical method of asymptotic homogenization describes the effect of wave dispersion by accounting for the influence of the first- and second-order terms on the asymptotic expansion for relatively long wavelengths in fiber reinforced composites as

* Corresponding author.
E-mail addresses: yswang@bjtu.edu.cn, yueshengwang@hotmail.com (Y.-S. Wang).

http://dx.doi.org/10.1016/j.euromechsol.2014.05.009
0997-7538/

shown by Parnell and Abrahams (2006, 2008). This approach fails when the observation time is relatively long or when the characteristic size of the perturbation is small if it is comparable to the representative volume element (unit cell).

The present work is related to the study of wave propagation in triclinic laminated composite materials with applications to damage detection and health monitoring for periodic laminated composites. The dispersive method is considered here for two spatial dimensions and the propagation of the wave is oblique to the layering. The present investigation is an extension of previous results reported at the literature (Vivar-Pérez et al., 2009) for one-dimensional case.

2. Method of dynamic homogenization

We consider an anisotropic elastic body of a periodic structure occupying a bounded region $\bigcirc$ in $\mathbb{R}^3$ space with Lipschitz boundary $\partial\bigcirc = \partial_1\bigcirc \cup \partial_2\bigcirc$ such that $\partial_1\bigcirc \cap \partial_2\bigcirc = \phi$ where $\partial_1\bigcirc$ and $\partial_2\bigcirc$ are boundary portions. It assumed that the region $\bigcirc$ is made up by periodic repetition of the unit cell Y in the form of a parallelepiped with dimensions $\varepsilon y_i (i = 1,2,3)$, where ε is the ratio of the unit cell size (i.e. period of the structure) to a typical length in the region. The method is presented for the particular case of anti-plane wave propagation in a periodically layered triclinic composite with two-phase materials see Fig. 1, where Γ^ε is the interface separating layer 1 and layer 2, and Γ is the interface in the unit cell. The medium is assumed to be layered in the x_1 direction, with all material parameters independent of x_2 and x_3. The perfect interface conditions are considered, i.e., the displacements and stresses are continuous at the interface (Chen and Fish, 2001).

The anti-plane problem is formulated in a bounded subset Ω^ε of $\mathbb{R}^2$, i.e. a boundary-value problem within a two-dimensional domain in the x_1x_2-plane. The reference cell is denoted by

$$Y = \left\{ \boldsymbol{y} = (y_1, y_2) \in \mathbb{R}^2 : 0 < y_i < l_i,\ i = 1,2 \right\},$$

where l_i are given positives numbers. Note that the subset Ω^ε is

$$\Omega^\varepsilon = \varepsilon Y = \left\{ \boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2 : \varepsilon^{-1} x_i \in Y,\ i = 1,2 \right\},$$

where $\boldsymbol{y} = \boldsymbol{x}/\varepsilon$.

The anti-plane problem is modeled mathematically in the form

$$\frac{\partial \sigma^\varepsilon_{3i}}{\partial x_i} - \rho^\varepsilon \frac{\partial^2 u^\varepsilon_3}{\partial t^2} = 0 \ \text{ in } \ \Omega^\varepsilon \times \left]0,\ \tau\right[, \tag{1a}$$

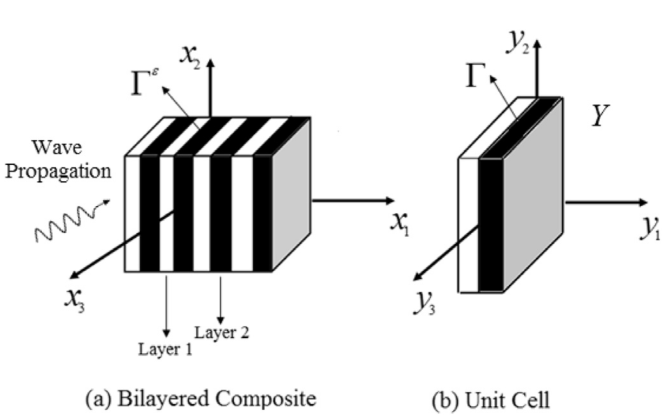

Fig. 1. The layered composite and the unit cell.

$$u^\varepsilon_3(\boldsymbol{x}, t)\big|_{t=0} = U(\boldsymbol{x}),\quad \frac{\partial u^\varepsilon_3(\boldsymbol{x}, t)}{\partial t}\bigg|_{t=0} = g(\boldsymbol{x}) \ \text{ in } \ \Omega^\varepsilon, \tag{1b}$$

$$u^\varepsilon_3(\boldsymbol{x}, t)\big|_{\partial_1\Omega^\varepsilon} = h(t),\ \sigma^\varepsilon_{3i} n_i\big|_{\partial_2\Omega^\varepsilon} = q(t) \text{ for } t > 0, \tag{1c}$$

$$[[u^\varepsilon_3]] = 0,\ [[\sigma^\varepsilon_{3i}]] n_i = 0 \text{ in } S^\varepsilon, \tag{1d}$$

where $\sigma^\varepsilon_{3i} = C_{3i3j}(x_1/\varepsilon)\, \partial u^\varepsilon_3/\partial x_j$ and $\partial\Omega^\varepsilon = \partial_1\Omega^\varepsilon \cup \partial_2\Omega^\varepsilon$ such that $\partial_1\Omega^\varepsilon \cap \partial_2\Omega^\varepsilon = \phi$; $[[\bullet]]$ denotes the difference of the values on the opposite sides of the unit cell Y (henceforth, the Latin indices take values 1 and 2); $C_{3i3j}(y_1) = C^{(1)}_{3i3j}$ for $0 < y_1 < \gamma l_1$ and $C_{3i3j}(y_1) = C^{(2)}_{3i3j}$ for $\gamma l_1 < y_1 < l_1$ (γ is the volume ratio of layer 1); S^ε is the projection of Γ^ε on x_1x_2-plane; and n_j is the unit vector in the outward normal direction.

2.1. Displacement formulation

Elimination of the stress in (1a) leads to the partial differential equation for u^ε_3

$$\frac{\partial}{\partial x_i}\left(C_{3i3j}(y_1) \frac{\partial u^\varepsilon_3}{\partial x_j} \right) - \rho(y_1) \frac{\partial^2 u^\varepsilon_3}{\partial t^2} = 0, \tag{2}$$

or, with consideration of $u^\varepsilon_3 = u_3(\boldsymbol{x}, \boldsymbol{y}, t) = v(\boldsymbol{x}, \boldsymbol{y})\, T(t)$, to the equivalent differential equations:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}t^2} + \omega^2 T = 0 \text{ for } t > 0, \tag{3a}$$

$$\mathrm{L}(v) + \rho(y_1)\omega^2 v(\boldsymbol{x}, \boldsymbol{y}) = 0 \text{ in } \Omega^\varepsilon, \tag{3b}$$

with the boundary conditions:

$$v = u_0 \text{ on } \partial_1\Omega^\varepsilon,\quad C_{3i3j}(y_1) \frac{\partial v(\boldsymbol{x}, \boldsymbol{y})}{\partial x_j} n_i = S_0 \text{ on } \partial_2\Omega^\varepsilon, \tag{4}$$

under the perfect contact conditions:

$$[[v]] = 0,\quad \left[\left[C_{3i3j} \frac{\partial v}{\partial x_j} \right]\right] n_i = 0 \text{ on } S^\varepsilon, \tag{5}$$

where

$$\mathrm{L}(v) = \frac{\partial}{\partial x_i}\left(C_{3i3j}(y_1) \frac{\partial v(\boldsymbol{x}, \boldsymbol{y})}{\partial x_j} \right),$$

is an elliptic differential operator in the domain Ω^ε; the prescribed functions are $u_0 \in H^1(\partial_1\Omega^\varepsilon)$; $\omega^2 \rho \boldsymbol{v} \in L^2(\Omega^\varepsilon)$; and $S_0 \in L^2(\partial_2\Omega^\varepsilon)$. The coefficients $C_{3i3j}(y_1)$ are bounded measurable functions satisfying the symmetry and ellipticity conditions. The weak solution $v(\boldsymbol{x}\boldsymbol{y})$ of problem (3b) and (4)–(5) exists and is unique (Oleinik et al., 1992).

Introducing the notation

$$\sigma_{3i}(\boldsymbol{x}, \boldsymbol{y}) = C_{3i3j}(y_1) \frac{\partial v(\boldsymbol{x}, \boldsymbol{y})}{\partial x_j},$$

Eq. (3b) can be written in the form

$$\frac{\partial \sigma_{3i}(\boldsymbol{x}, \boldsymbol{y})}{\partial x_i} + \omega^2 \rho(y_1) v(\boldsymbol{x}, \boldsymbol{y}) = 0. \tag{6}$$

Taking into account a regular asymptotic expansion of the circular frequency ω and following the literature (Sanchez-Palencia,