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Propagation and attenuation of Rayleigh waves in a partially-saturated porous solid with impervious boundary

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ABSTRACT

Propagation of Rayleigh waves is studied in a three-phase porous solid half-space, which is bounded above by an impervious plane surface. In this dissipative medium, Rayleigh wave propagates as an inhomogeneous wave, which decays with distance from the stress-free plane boundary. The impervious boundary restricts the flow of pore-fluids to the interior of porous solid only. This is ensured by fixing the fluid-pressure gradient in pores at boundary or with the sealing of surface pores. In either case, the existence and propagation of inhomogeneous wave are represented by a dispersion equation, which happens to be complex and irrational. This equation is rationalized into an algebraic equation of degree 24, which is solved for a numerical example. Solutions of the dispersion equation are checked to represent an inhomogeneous wave decaying with depth. Each qualified solution is resolved to define the phase velocity and attenuation coefficient of a Rayleigh wave in the medium. Numerical example compares the velocity and attenuation of Rayleigh wave in porous sandy loam for the two representations of the impervious boundary, one with sealed pores and other with no fluid-pressure gradient. Effects of saturation degree, porosity, capillary pressure, pore-fluids viscosity and frame anelasticity are observed on the propagation characteristics of Rayleigh waves. Existence of second Rayleigh wave is checked numerically. Such a wave is possible only when the porous frame is highly anelastic and saturated enough.

1. Introduction

Connected pores are ubiquitous in every geological medium that consists of a rock or soil. The pore space may be occupied by the mixture of a liquid and a rarefied gas. Such a twin-fluid mixture in pores represents the partial saturation, when the liquid fills only a part of the connected pore space and the bubbles of a rarefied gas span the remaining void space. In other words, a poroelastic solid saturated with two-phase viscous fluid represents a fairly realistic model for sedimentary or reservoir rocks. Then, the study of elastic waves in partially-saturated porous media may be of great interest in the exploration of subsurface resources. Biot (1956, 1962a,b) formulated the dynamical equations for the propagation of elastic waves in a poroelastic solid saturated completely with a single-phase fluid. Berryman et al. (1988) considered the extension of Biot's single pore-fluid formulation to the porous media saturated by multiple fluids. In another extension of Biot's theory, Pride and Berryman (2003a,b) considered fluid transport mechanism and used volume averaging technique to derive the governing equations for fluid-saturated double porosity media. Pride et al. (2004) studied a mesoscale model, which was based on the heterogeneity in the pore-fluid type, that is, patchy saturation. In this model, a strong contrast between the bulk moduli of two immiscible pore-fluids is held responsible for the large attenuation observed in seismograms. Another approach does not need the description of pore structure and, hence, seems to be more convenient than the extensions of Biot's theory. Based on mixture theory, this approach assumes the uniform existence of non-interacting constituent phases. Starting with Brutsaert (1964), an extensive survey of earlier literature on mixture theory is given by Bowen (1976). The comprehensive procedures relevant to the wave propagation in porous solids saturated with multiphase fluids are found in Bedford and Drumheller (1983), Garg and Nayfeh (1986), Santos et al. (1990a,b) and Corapcioglu and Tuncay (1996). A mathematical model presented recently by Lo et al. (2005) is also based on continuum mixture theory and accounts for the changes in capillary pressure and the viscous/inertial coupling among constituents.

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Body waves in a dissipative medium lose energy in the interior and induce only minor disturbances near the surface. Surface waves, however, have the potential to cause destructive vibrations to structures and buildings. Propagation and attenuation of the surface waves in liquid-saturated porous media are of increasing interest in the disciplines of hydrogeology and geomechanics, notably in borehole logging and onshore/offshore civil engineering. Jones (1961) studied the propagation of Rayleigh waves in porous solids saturated with single fluid. In this study, a mathematically incorrect approach was used to explain the existence of two different Rayleigh waves. Using the same approach, Lo (2008) studied the propagation of Rayleigh waves in a porous solid saturated with two-phase fluid and suggested the existence of three different Rayleigh waves in unsaturated porous medium. The present author wrote a comment (Sharma, 2012a) on the procedure followed by Lo (2008) and then studied the same problem in correct mathematics (Sharma, 2012b), using a different formulation (Sharma and Kumar, 2011). A complex irrational dispersion equation was derived for the propagation of Rayleigh waves in partially-saturated porous solid with pervious (fully opened surface pores) boundary. This dispersion equation was solved, numerically, after rationalizing into a polynomial form (Currie et al., 1977). The complex roots of the dispersion equation, which qualify to represent Rayleigh waves, were resolved to define phase velocities and quality factors for existing Rayleigh waves.

Present work is a continuation of the earlier work (Sharma, 2012b), i.e., Rayleigh waves in partially-saturated porous medium with pervious boundary. Pervious boundary is replaced with an impervious one. Impervious boundary of the porous medium is represented through the sealing of surface pores (Deresiewicz and Skalak, 1963) or by the gradient of fluid-pressures in surface pores (Tajuddin, 1984). Different conditions at the boundary of three-phase medium result in two dispersion equations for the propagation of inhomogeneous waves. The complex irrational dispersion equations are rationalized to polynomial forms. In either case, the resulting algebraic equation of degree 24 can be solved exactly through numerical methods. The roots, which qualify to represent the propagation of Rayleigh waves, are identified. Each such root is resolved to calculate the propagation velocity and quality factor of a Rayleigh wave in partially-saturated porous medium with impervious boundary. In this study, skeleton refers to drained porous frame and porous solid refers to composite porous aggregate saturated by gas—liquid mixture. The words unsaturated and anelastic are synonyms for partially-saturated and viscoelastic, respectively.

2. Basic equations

A partially-saturated porous solid is assumed to be a continuum consisting of solid skeleton with connected void space occupied by the mixture of a gas and a liquid. The indices '*s*', '*g*', '*l*', are used to identify the three constituents of composite porous medium, i.e., solid grains, pore-gas and pore-liquid respectively. In porous aggregate with total connected porosity (f), volume fractions of the constituents are defined as

$$\delta_s = 1 - f, \quad \delta_g = (1-\sigma)f, \quad \delta_l = \sigma f, \tag{1}$$

where σ is the fraction of liquid in pore-fluids mixture. Wave motion in the composite medium is modelled through the mixture theory (Tuncay and Corapcioglu, 1997), which assumes that the pore-size should be very small as compared to the wave-length. Following Lo et al. (2005), the equations of motion for low-frequency vibrations of constituent particles in isotropic porous solid, in the absence of body forces, are given by

$$\delta_s \tau^{(p)}_{ij,j} = \delta_s \rho_s \ddot{u}_i - q_g(\dot{v}_i - \dot{u}_i) - q_l(\dot{w}_i - \dot{u}_i),$$
$$-\delta_g p^{(g)}_{,i} = \delta_g \rho_g \ddot{v}_i + q_g(\dot{v}_i - \dot{u}_i), \ -\delta_l p^{(l)}_{,i} = \delta_l \rho_l \ddot{w}_i + q_l(\dot{w}_i - \dot{u}_i), \tag{2}$$

where $p^{(g)}$,$p^{(l)}$ are pressures in fluid phases and $\tau^{(p)}_{ij}$ is the stress tensor for drained porous frame. ρ's are intrinsic densities of the constituents. u_i, v_i and w_i denote the components of displacements of solid, gas and liquid particles, respectively. Dot over a variable implies partial derivative with time and comma before an index implies partial space differentiation. A repetition of index (subscript) implies summation. Darcy's law relates viscous dissipation to the motion of gas and liquid particles relative to pore-walls. Dissipation coefficients for gas (q_g) and liquid (q_l) are defined as follows.

$$q_k = \eta_k \delta_k^2 \Big/ (\chi \chi_k), \quad (k = g, l), \tag{3}$$

where η_k and χ_k define viscosity and relative permeability of the fluid phase k. χ denotes intrinsic permeability of the porous medium.

Constitutive relations for stresses in porous skeleton and fluid-pressures in pore-space are given by Tuncay and Corapcioglu (1997)

$$\delta_s \tau^{(p)}_{ij} = (a_{11}\nabla\cdot\mathbf{u} + a_{12}\nabla\cdot\mathbf{v} + a_{13}\nabla\cdot\mathbf{w})\delta_{ij} + G_p\left(u_{i,j} + u_{j,i} - \frac{2}{3}u_{k,k}\delta_{ij}\right),$$
$$-\delta_g p^{(g)} = (a_{21}\nabla\cdot\mathbf{u} + a_{22}\nabla\cdot\mathbf{v} + a_{23}\nabla\cdot\mathbf{w}),$$
$$-\delta_l p^{(l)} = (a_{31}\nabla\cdot\mathbf{u} + a_{32}\nabla\cdot\mathbf{v} + a_{33}\nabla\cdot\mathbf{w}), \tag{4}$$

where δ_{ij} is Kronecker symbol and G_p denotes rigidity of the medium. Elastic coefficients a_{ij} are derived in appendix.

3. Rayleigh waves

In Cartesian coordinate system (x,y,z), porous solid occupies the half-space $z > 0$, bounded by the plane $z = 0$. Isotropy in this medium allows to study wave motion in the x-z plane without losing any information. Hence, all the quantities become independent of the y-coordinate. Surface of the porous solid is considered to be free of any resultant force. Following Sharma (2012b), the displacement potentials ϕ_j,($j = 1$,2,3), represent the propagation of three dilatation (P_1,P_2,P_3) waves with velocities α_1,α_2,α_3, respectively. The fourth potential (ϕ_4) represents the propagation of shear wave in x-z plane (i.e., *SV* wave) with velocity β. Expressions for the velocities of four waves are given in appendix. For the Helmholtz resolution, in-plane displacements of solid and fluid phases are given by

$$u_x = \sum_{j=1}^{3}\frac{\partial\phi_j}{\partial x} + \frac{\partial\phi_4}{\partial z}, \ v_x = \sum_{j=1}^{3}\mu_j\frac{\partial\phi_j}{\partial x} + \mu_4\frac{\partial\phi_4}{\partial z}, \ w_x = \sum_{j=1}^{3}\nu_j\frac{\partial\phi_j}{\partial x} + \nu_4\frac{\partial\phi_4}{\partial z},$$
$$u_z = \sum_{j=1}^{3}\frac{\partial\phi_j}{\partial z} - \frac{\partial\phi_4}{\partial x}, \ v_z = \sum_{j=1}^{3}\mu_j\frac{\partial\phi_j}{\partial z} - \mu_4\frac{\partial\phi_4}{\partial x}, \ w_z = \sum_{j=1}^{3}\nu_j\frac{\partial\phi_j}{\partial z} - \nu_4\frac{\partial\phi_4}{\partial x}, \tag{5}$$

where μ_j and ν_j are explained in appendix. Expressions (5) are used in relations (4) to calculate stresses and fluid-pressures.

For the harmonic plane waves propagating in x–z plane, the displacement potentials are chosen as follows.

$$\phi_j = A_j e^{\imath k(x-ct) - kd_j z}, \quad (j = 1, 2, 3, 4), \tag{6}$$