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A constitutive model for interface problems with frictional contact and cohesion

A. Oinonen*, G. Marquis

Aalto University School of Engineering, Department of Applied Mechanics, P.O. Box 14300, FIN-00076 Aalto, Espoo, Finland

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ABSTRACT

A numerical approach to modelling contact problems with a unified friction and cohesion interface is formulated. A new nonlinear friction law is suggested for modelling micro-slip of metallic junctions due to contact asperities, and an associated cohesive zone due to adhesion describes the linear portion of the unified total interfacial hysteresis. A variational equality including both the regularized friction and cohesion terms is formulated for the numerical solution of the derived boundary value problem. The suggested modelling technique is readily implementable in the finite element method. There is an application on the representative problem involving the adhesively bonded and significantly normal-stressed contact surface. Macroscopic constitutive relationships between the cyclic tangential load and micro-displacements are established for the set of constant compressive normal loads. The related micromechanical arguments and experimental observations supporting the modelling theory are addressed.

1. Introduction

Recently, adhesively reinforced frictional interfaces (RFI) have been the focus of both experimental research and development of modelling of mechanical responses (Dragoni and Mauri, 2000, 2002; Hurme et al., 2011; Oinonen and Marquis, 2011a, 2011b; Castagnetti and Dragoni, 2012, 2013). There are many potential engineering applications of RFIs, e.g. cylindrical fits (Dragoni and Mauri, 2002) and bolted/bonded (hybrid) lap-joints (Albrecht and Sahli, 1988). Depending on the application, different adhesive types, such as acrylics (Albrecht and Sahli, 1988), anaerobic retainers (Dragoni and Mauri, 2000, 2002) and epoxies (Hurme et al., 2011; Oinonen and Marquis, 2011a, 2011b), are used for RFIs. A microstructure of bonded interfaces consists of a small amount of adhesive material inside the micro-volumes between compressed contact surfaces (Dragoni and Mauri, 2000; Oinonen and Marquis, 2011b). Opposite to conventional bonded joints, a continuous layer of adhesive does not exist for strongly clamped RFI-based connections (Dragoni and Mauri, 2000; Oinonen and Marquis, 2011b). The research on RFIs is particularly important for more advanced design of light weight structures in high strength steel (HSS) involving hybrid lap-joined thin plate members and tubular hollow sections. Detailed constitutive models are needed for computational strength analysis of hybrid joints, since adhesively RFIs show sensitivity to overloads in the shear mode (Hurme et al., 2011).

Physical arguments related to computational modelling of nonlinear friction have previously been addressed e.g. by Oden and Pires (1983) and Anand (1993). A general case of the frictional contact problem can numerically be solved from the variational form of the nonlinear boundary value problem (BVP) involving both the Signorini's complementary conditions and friction law (Rabier and Oden, 1987; Wriggers, 1995; Radi et al., 1998). Oden and Pires (1983) developed a macroscopic model of the mechanical contact with friction and presented variational principles of the problem. Several variations of the nonlocal micro-slip friction laws have been implemented in the finite element method (FEM) (Fredriksson, 1976; Campos et al., 1982; Simo and Laursen, 1992; Sellgren and Olofsson, 1999).

The concept of cohesive zones (CZ) was originated by Dugdale (1960) and further developed by Barenblatt (1962). Cohesive zone models (CZMs) involving pre-defined cohesive surfaces (Needleman, 1987) are used in computational strength analysis of composites (Camanho et al., 2003) and epoxy RFIs (Oinonen and Marquis, 2011a). Constitutive models and numerical procedures which couple the cohesive and frictional properties have previously been suggested and formulated by Raous et al. (1999), Döbert et al.

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* Corresponding author. Tel.: +358 503611709.
E-mail addresses: ahti.oinonen@aalto.fi (A. Oinonen), gary.marquis@aalto.fi (G. Marquis).

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Nomenclature

$a(\cdot,\cdot), f(\cdot)$	bilinear and linear forms
$j(\cdot), j_\varepsilon(\cdot)$	interface functional and regularized interface functional
g	gap in the direction of the normal to the contact surface
h	size of the finite element
n	dimension of the vector space
n_t	number of the accumulated loading cycles
q	pre-defined uniform tightening pressure of the test specimen
s^s, s^c	directions of motion at the driving surface and at the contact surface
t, Δt	pseudo time and time discretization parameter
t_t^a, t_t^{cr}	applied and critical tangential tractions due to cohesion
u_n, u_t	normal displacement and magnitude of tangential displacements
u_t^a, u_t^{cr}, $u_t^{\max}$	applied, critical and maximum tangential displacement amplitudes
u_h, u_{ref}	boundary and reference values of the tangential displacement
v_{cyc}	constant driving speed
w_n, w_t	normal and tangential virtual velocities at the contact interface
N, T	normal and tangential forces
T^c, T^f	shear forces due to cohesion and friction
T^{max}	maximum applied tangential force due to cohesion
ε_t^a, ε_t^{cr}, $\varepsilon_t^{\min}$	applied, critical and minimum tangential regularization parameters
κ_t	tangential stiffness due to the cohesive contribution at the interface
μ, μ_{cyc}	macroscopic and cyclic friction coefficients
σ_n, σ_t	normal and tangential pressures at the boundary
σ_0^a	average yield stress of the clamped contact junctions
τ, τ_{ref}	solved and measured macroscopic total shear stresses at the interface
τ_0, τ_0^c	ultimate shear stresses of the hybrid interface and adhesive material
φ_{cyc}^c, φ_{cyc}^f	cyclic regularization functions of the cohesion and friction functionals
ψ	normalizing weight parameter of the mollifier function φ_{cyc}^f
Ω, $\partial\Omega$	domain and the associated boundary of the domain
$\mathbf{h}$	logarithmic strain tensor
$\mathbf{n}$	unit vector outward and normal to the boundary
$\mathbf{t}$	traction vector with respect to the reference configuration
$\mathbf{t}_n$, $\mathbf{t}_t$	normal and tangential traction vectors
$\mathbf{u}(\mathbf{x},t)$	displacement field with respect to the reference configuration
$\mathbf{u}_n$, $\mathbf{u}_t$	normal and tangential displacement vectors
$\mathbf{u}_0$, $\boldsymbol{\sigma}_0$	initial states of the displacement, and stress due to the clamping load
$\mathbf{v}(\mathbf{x},t)$	velocity field with respect to the reference configuration
$\mathbf{w}$, $\mathbf{v}$	test functions
$\mathbf{x}$	vector of spatial coordinates
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\mathbf{R}^n$	vector space
$\mathbf{V}$, $\mathbf{W}$	test spaces of admissible displacements and velocities

(2000), Oinonen and Marquis (2011a) and Snozzi and Molinari (2013). For modelling of RFIs with a high normal pre-load, it is a convenient approach to separately determine the CZM, i.e. independently of the associated frictional contact model (Oinonen and Marquis, 2011a, 2011b). Assuming that overloads do not occur, a linear CZM is adequate for modelling cyclic responses due to the cohesive contribution, whereas the nonlinear friction law describes the associated micro-slip phenomena of the RFI.

Experimentally measured responses are valuable references especially for the more complex bonded interfaces due to a small amount of existing test data and still limited knowledge on mechanical behaviour of RFIs. The reference data should be based on measured average displacements over opposite surfaces of the specimen pair containing a sufficiently large amount of microstructural heterogeneities (Hill, 1972). Fouvry et al. (1995) measured mechanical responses and determined the shapes of micro-slip hysteresis for non-bonded steel surfaces subjected to cyclic loading. Hurme et al. (2011) tested ground-contact surfaces in HSS to determine macroscopic responses and shear strength properties of epoxy RFIs under cyclic shear loading. In the recent studies at Aalto University (Hurme et al., 2011; Oinonen and Marquis, 2011a, 2011b), the classical test of a napkin ring type (De Bruyne, 1962) was redesigned for applications involving significant axial pre-loads. A ratio of the annular contact area and outer radius of the napkin ring specimen is very small, and therefore a uniform interfacial stress distribution due to applied torsion load can certainly be assumed. Therefore, the adapted testing procedure (Hurme et al., 2011) provides good reference results for determining the macroscopic continuum behaviour originally based on the heterogeneous bonded microstructure of RFIs.

In engineering computations, it is generally assumed that the microstructure can be modelled as a continuous medium even though it is heterogeneous and contains voids. Macroscopic models of the interfacial response are as well computationally realistic implementations. A homogenized response of the body can be numerically solved and described on the concept of a representative volume element (RVE), according to Hill (1963, 1972). Under the assumptions of micro equilibrium and continuity of displacements, the macroscopic stress and strain quantities over the RVE are the statistical mean values of the respective local values (Hill, 1963, 1972; Stupkiewicz, 2007). In the FEM environment, the interfacial response of the RFI can be accurately reproduced by the volume averaging method (Hill, 1963) on a simple RVE that presents a short section of the napkin ring specimen (Oinonen and Marquis, 2011a). Even for the simple RVE, the high frictional stresses are induced at the contact corners while a stress field at the middle of the RVE is more uniform. The applicability of both the macroscopic interface model and numerical modelling procedure, initially tested on the RVE, have then been validated for computations on the larger scale FE models.

The objective of this paper is to develop constitutive relationships for modelling macroscopic responses of RFIs subject to cyclic shear loading. In particular, computationally efficient modelling of hysteresis of adhesively RFIs is studied without exceeding a damage threshold. A well-defined two-dimensional (2D) model problem is implemented in the FEM and a comparison of FE solutions is made against the experimental reference data produced by Hurme et al.