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Three-dimensional thermal stress analysis of laminated composite plates with general layups by a sampling surfaces method

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ABSTRACT

A paper focuses on the application of the method of sampling surfaces (SaS) to three-dimensional (3D) steady-state thermoelasticity problems for orthotropic and anisotropic laminated plates subjected to thermal loading. This method is based on selecting inside the nth layer I_n not equally spaced SaS parallel to the middle surface of the plate in order to choose temperatures and displacements of these surfaces as basic plate variables. Such an idea permits the presentation of the proposed thermoelastic laminated plate formulation in a very compact form. It is worth noting that the SaS are located inside each layer at Chebyshev polynomial nodes that leads to a uniform convergence of the SaS method. As a result, the SaS method can be applied efficiently to the 3D stress analysis of cross-ply and angle-ply composite plates with a specified accuracy utilizing the sufficient number of SaS.

1. Introduction

Three-dimensional (3D) exact analysis of laminated composite plates has attracted the considerable attention during past forty years. This is due to the fact that the validity of approximate plate theories and plate finite elements can be assessed by comparing their predictions with 3D exact solutions. The analytical solution of elasticity for a simply supported isotropic rectangular plate was presented by Vlasov (1957). The extensions of Vlasov's solution to orthotropic laminated plates were done by Pagano (1969, 1970a), and Srinivas and Rao (1970). Murakami (1993) generalized the work of Pagano (1969) to the cylindrical bending of simply supported laminates subjected to thermal loading. Tungikar and Rao (1994), Noor et al. (1994) and Bhaskar et al. (1996)) derived 3D exact solutions for laminated cross-ply rectangular plates subjected to thermomechanical loads. Tauchert (1980) gave 3D exact solutions of thermoelasticity for simply supported orthotropic laminates using the method of displacement potentials. The analytical solutions for functionally graded single-layer and laminated plates under mechanical and thermal loads were derived by Cheng and Batra (2000), Reddy and Cheng (2001), Vel and Batra (2002), Kashtalyan (2004), and Woodward and Kashtalyan (2011).

Pagano (1970b) presented the exact solution for the cylindrical bending of laminated composite plates with general layups. The response of a thermoelastic anisotropic laminated plate in cylindrical bending was investigated analytically by Bhaskar et al. (1996), and Vel and Batra (2001). The developments for antisymmetric angle-ply laminates in the framework of the 3D theory were carried out by Noor and Burton (1990), Savoia and Reddy (1992, 1995), and Kulikov and Plotnikova (2012a). However, the reliable 3D solutions for thermoelastic laminated composite plates of general lay-up configurations can not be found in the current literature. Partially, the present paper serves to fill the gap of knowledge in this research area.

To solve such a problem, we invoke the efficient method of sampling surfaces (SaS) developed recently by Kulikov and Plotnikova (2012a, 2012b, 2013, 2014) for the analysis of orthotropic and anisotropic laminated plates and shells. As SaS denoted here by $\Omega^{(n)1}, \Omega^{(n)2}, \ldots, \Omega^{(n)I_n}$, we choose outer surfaces and any inner surfaces inside the nth layer of the plate and introduce temperatures $T^{(n)1}, T^{(n)2}, \ldots, T^{(n)I_n}$ and displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, \ldots, \mathbf{u}^{(n)I_n}$ of these surfaces as basic plate variables, where I_n is the total number of SaS chosen for each layer ($I_n \geq 3$). Such choice of temperatures and displacements with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer permits the presentation of governing equations of the thermal laminated plate formulation in a very compact form. It is necessary to note that the term SaS should not be confused with such terms as fictitious interfaces or mathematical interfaces, which are extensively used in layer-wise theories. The main difference consists in the lack of possibility to employ polynomials of high degree in the thickness direction because in conventional layer-wise thermal shell theories only third and fourth order polynomial interpolations are admissible

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(see, e.g. Carrera, 2000, 2005; Carrera and Ciuffreda, 2004; Robaldo et al., 2005; Robaldo and Carrera, 2007). This restricts the use of the fictitious/mathematical interfaces technique for the 3D thermal stress analysis of thick laminated composite plates. On the contrary, the SaS method allows the use of Lagrange polynomials of *high* degree. This fact gives the opportunity to derive the 3D solutions for laminated composite plates with a prescribed accuracy employing a sufficiently large number of not equally spaced SaS.

It is worth noting that the developed approach with the arbitrary number of equally spaced SaS (Kulikov and Plotnikova, 2011) does not work properly with the Lagrange polynomials of high degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of Chebyshev polynomial nodes (Burden and Faires, 2010) inside each layer can help to improve significantly the behavior of the Lagrange polynomials of high degree because such a choice allows one to minimize uniformly the error due to Lagrange interpolation.

The origins of using the SaS can be found in contributions of Kulikov (2001) and Kulikov and Carrera (2008) in which three, four and five equally spaced SaS are employed. The SaS method with the arbitrary number of equispaced SaS is considered by Kulikov and Plotnikova (2011). The more general approach with the SaS located at Chebyshev polynomial nodes has been developed later (Kulikov and Plotnikova, 2012a, 2012b).

2. Description of temperature field

Consider a laminated plate of the thickness h. Let the middle surface Ω be described by Cartesian coordinates x_1 and x_2. The coordinate x_3 is oriented in the thickness direction. The transverse coordinates of SaS inside the nth layer are defined as

$$x_3^{(n)1} = x_3^{[n-1]}, \quad x_3^{(n)I_n} = x_3^{[n]},$$
$$x_3^{(n)m_n} = \frac{1}{2}\left(x_3^{[n-1]} + x_3^{[n]}\right) - \frac{1}{2} h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \tag{1}$$

where $x_3^{[n-1]}$ and $x_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ (Fig. 1); $h_n = x_3^{[n]} - x_3^{[n-1]}$ is the thickness of the nth layer; I_n is the number of SaS corresponding to the nth layer; the index n identifies the belonging of any quantity to the nth layer and runs from 1 to N; N is the total number of layers; the index m_n identifies the belonging of any quantity to the inner SaS of the nth layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n to be introduced later for describing all SaS of the nth layer run from 1 to I_n. Besides, the tensorial indices i, j, k, l range from 1 to 3 and Greek indices α, β range from 1 to 2.

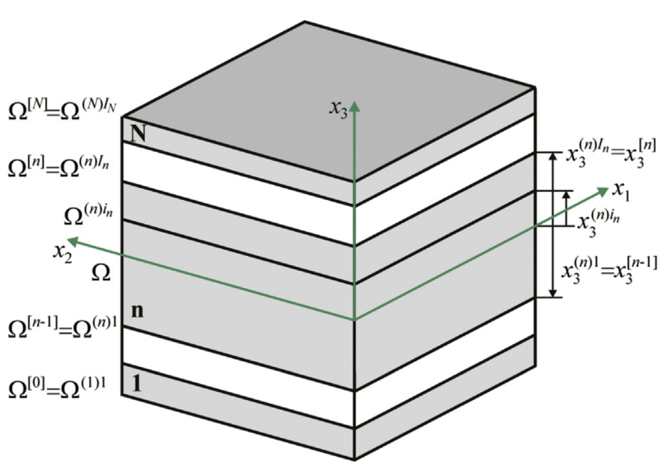

Fig. 1. Geometry of the laminated plate.

Remark 1. The transverse coordinates of inner SaS (1) coincide with the coordinates of Chebyshev polynomial nodes (Burden and Faires, 2010). This fact has a great meaning for a convergence of the SaS method (Kulikov and Plotnikova, 2012a, 2012b).

The relation between the temperature T and the temperature gradient $\mathbf{\Gamma}$ is given by

$$\mathbf{\Gamma} = \nabla T. \tag{2}$$

In a component form, it can be written as

$$\Gamma_i = T_{,i}, \tag{3}$$

where the symbol $(\ldots)_{,i}$ stands for the partial derivatives with respect to coordinates x_i.

We start now with the first fundamental assumption of the proposed thermoelastic laminated plate formulation. Let us assume that temperature and temperature gradient fields are distributed through the thickness of the nth layer as follows:

$$T^{(n)} = \sum_{i_n} L^{(n)i_n} T^{(n)i_n}, \quad x_3^{[n-1]} \le x_3 \le x_3^{[n]}, \tag{4}$$

$$\Gamma_i^{(n)} = \sum_{i_n} L^{(n)i_n} \Gamma_i^{(n)i_n}, \quad x_3^{[n-1]} \le x_3 \le x_3^{[n]}, \tag{5}$$

where $T^{(n)i_n}(x_1, x_2)$ are the temperatures of SaS of the nth layer $\Omega^{(n)i_n}$; $\Gamma_i^{(n)i_n}(x_1, x_2)$ are the components of the temperature gradient at the same SaS; $L^{(n)i_n}(x_3)$ are the Lagrange polynomials of degree $I_n - 1$ defined as

$$T^{(n)i_n} = T\left(x_3^{(n)i_n}\right), \tag{6}$$

$$\Gamma_i^{(n)i_n} = \Gamma_i\left(x_3^{(n)i_n}\right), \tag{7}$$

$$L^{(n)i_n} = \prod_{j_n \ne i_n} \frac{x_3 - x_3^{(n)j_n}}{x_3^{(n)i_n} - x_3^{(n)j_n}}. \tag{8}$$

The use of relations (3), (4), (6) and (7) yields

$$\Gamma_\alpha^{(n)i_n} = T_{,\alpha}^{(n)i_n}, \tag{9}$$

$$\Gamma_3^{(n)i_n} = \sum_{j_n} M^{(n)j_n}\left(x_3^{(n)i_n}\right) T^{(n)j_n}, \tag{10}$$

where $M^{(n)j_n} = L_{,3}^{(n)j_n}$ are the derivatives of Lagrange polynomials, which are calculated at SaS as follows:

$$M^{(n)j_n}\left(x_3^{(n)i_n}\right) = \frac{1}{x_3^{(n)j_n} - x_3^{(n)i_n}} \prod_{k_n \ne i_n, j_n} \frac{x_3^{(n)i_n} - x_3^{(n)k_n}}{x_3^{(n)j_n} - x_3^{(n)k_n}} \quad \text{for } j_n \ne i_n,$$
$$M^{(n)i_n}\left(x_3^{(n)i_n}\right) = -\sum_{j_n \ne i_n} M^{(n)j_n}\left(x_3^{(n)i_n}\right). \tag{11}$$

It is seen from Eq. (10) that the transverse component of the temperature gradient $\Gamma_3^{(n)i_n}$ is represented as a *linear combination* of temperatures of all SaS of the nth layer $T^{(n)j_n}$.