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Determination of the shakedown limit for large, discrete frictional systems

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ABSTRACT

It has been shown that complete frictional contacts subjected to cyclic loads can *shake down* such that the interface becomes fully stuck after some number of cycles (even if partial-slip occurs at the onset of loading). However, if the amplitude of the cyclic load is greater than a particular value (the *shakedown limit*), it is impossible for shakedown to occur. In this paper, we examine a numerical approach for determining the shakedown limit for elastic frictional systems subjected to quasi-static loads using a discrete formulation. We then use this technique to determine the shakedown limit for a finite element model of a (coupled) complete contact with ~50,000 total degrees of freedom and ~250 along the contact interface. Finally, we compare the calculated value of the shakedown limit to a series of over 1000 transient simulations and investigate the influence of initial conditions on steady state frictional energy dissipation. The results demonstrate that the dissipative properties of complete contacts can be highly dependent on the initial residual slip displacement state.

1. Introduction

It is extremely common for frictional contacts in engineering systems to be subjected to a combination of a static load (to form the contact) and a time varying cyclic load (e.g. due to vibrations). Some examples include riveted and bolted joints (Abad et al., 2012; Law et al., 2006), dove-tail connections in jet engines (Rajasekaran and Nowell, 2006; Xi et al., 2000), and spline couplings (Limmer et al., 2001; Banerjee and Hills, 2006; Banerjee et al., 2008) to name just a few. Furthermore, it is well known that even when the contact interface appears nominally to be stuck, these loading conditions frequently result in small zones of *micro-slip* (or *partial-slip*) along the contact interface (where the relative tangential displacement is generally of the order 25–100 μm (Szolwinski and Farris, 1996)). The effect of micro-slip on system performance is two-fold: on the one hand, it results in the well known damage process of *fretting fatigue*, which significantly reduces the service life of components (Farris et al., 2000); but on the other hand, it provides a source of damping, which can be beneficial for system performance (Law et al., 2006).

Much of the previous work on micro-slip and fretting fatigue has focused on *incomplete* (or *non-conforming*) contacts (e.g. (Nowell and Hills, 1987; Hills et al., 2012)), which are characterized by the edge of contact being smooth such that the bodies do not 'conform' in the absence of applied loads. This is probably due to their relevance in engineering practice but also because closed form solutions exist for many relevant loading cases, e.g. (Mindlin and Deresiewicz, 1953; Jäger, 1998; Ciavarella, 1998). For these contacts, the contact area is a function of the applied normal load, and the contact pressure distribution smoothly tends to zero as the contact edge is approached. As a direct consequence of this, a zone of micro-slip will almost always arise at the edge of an incomplete contact that is subjected to cyclic loads (Barber et al., January–February 2008).

Complete (or *conforming*) contacts, such as the contact shown in Fig. 1, are characterized by the two contacting bodies 'conforming' in the absence of the applied loads. For this class of contacts, the edge of contact is sharp, and the contact area either does not change in the presence of applied loads or becomes smaller (due to the edges 'lifting off').[1] Because sharp features result in a large

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[1] For this reason, this type of contact is sometimes referred to as a *receding* contact (Dundurs and Stippes, 1970).

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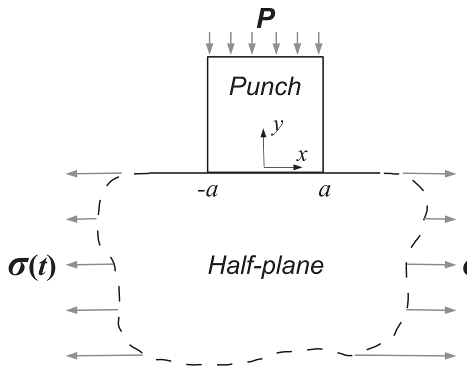

Fig. 1. The complete contact formed between a square elastic punch and an elastically similar half-plane showing: the static normal load, P, the time-varying load, $\sigma(t)$, the size of the contact, $2a$, and the (x,y) coordinate set.

stress concentration in their vicinity, complete contacts are usually avoided when possible. However, these contacts do arise in some critical applications such as in the spline couplings that join the central shafts in jet engines (Limmer et al., 2001; Banerjee and Hills, 2006).

Numerical analysis of complete contacts subjected to cyclic loads sometimes predicts that the contact *shakes down* (Churchman and Hills, June 2006; Banerjee and Hills, 2006): that is, after the first few loading cycles, a favourable residual displacement state is developed that inhibits slip for all subsequent cycles. This behaviour clearly has some similarities to cyclic plasticity problems, which has prompted the question as to whether a theorem analogous to Melan's plastic shakedown theorem (Melan, 1936) could be developed for frictional contacts. In other words, if there exists a residual displacement distribution that would inhibit slip at all times during load cycle (i.e. a *safe* displacement distribution), is the existence of this displacement distribution sufficient to guarantee that the contact will shake down?

This question has been answered unequivocally by Klarbring et al. (December 2007) for discrete contact problems and by Barber et al. (January–February 2008) for continuum problems. These authors have proven that the existence of a safe displacement distribution is both a necessary and sufficient condition to guarantee shakedown if the contact is *uncoupled*: that is, if relative tangential contact displacements have no influence on the distribution of normal contact traction. Thus, for uncoupled contacts, there exists a load factor, i.e. a ratio of cyclic load amplitude to static load (e.g. σ/P in Fig. 1), above which shakedown is impossible and below which it is guaranteed; hence the contact's steady state response never depends on initial conditions.

On the other hand, if the contact is coupled, the existence of a safe displacement distribution is still a necessary condition for shakedown to be possible, but it is no longer sufficient. In fact, for coupled contacts, there almost always exists a range of load factors for which the steady state response depends on the initial residual displacement distribution (Klarbring et al., December 2007; Barber et al., January–February 2008). Ahn et al. (December 2008) have demonstrated this behaviour for a two-node system and have shown that for coupled contacts there exists:

1. a load factor below which shakedown is guaranteed to occur, denoted λ_1
2. a load factor above which shakedown is impossible, denoted λ_2.

Note that these two limits on the applied load may be zero for many contacts, i.e. $\lambda_1 = \lambda_2 = 0$, (e.g. for incomplete contacts), implying that shakedown is impossible. Moreover, even if $\lambda_2 > 0$, it is still possible that $\lambda_1 = 0$, implying that although shakedown is sometimes possible, whether it actually occurs is always dependent on initial conditions.

Ahn et al. (December 2008) also presented a method for calculating both λ_1 and λ_2. Unfortunately, their approach involves solving a system of equations that becomes combinatorially more complex as the number of degrees of freedom in the system increases. For instance, Jang and Barber (2011) were able to apply this approach to determine both λ_1,λ_2 for a system of 10 cracks, but applying this analysis to a significantly larger frictional system would be computationally prohibitive. As it is common for the finite element models used in practice to incorporate one or two orders of magnitude more nodes along the frictional interface than are considered by Jang and Barber (2011), it is of practical interest to develop a more computationally efficient approach.

Björkman and Klarbring (1987) have presented an efficient approach for calculating λ_2, which we will henceforth refer to as the shakedown limit. To do this, these authors formulate the problem in discrete form, and show that the shakedown limit calculation can be posed as a Linear Programming problem for which several efficient solution algorithms exist. These authors then apply this calculation to two example problems, and also compare the results to numerical results obtained using the finite element method.

The aim of this paper is: (i) to re-visit the Linear Programming approach for calculating the shakedown limit and (ii) to examine the influence of initial conditions on the level of frictional dissipation present in the steady state. This is of practical importance because energy dissipation is associated with damage processes such as fretting fatigue and wear, which are themselves linked to component life. In particular, we wish to investigate whether energy dissipation is significantly influenced by initial conditions or if their effect can safely be neglected.

The structure of the paper is as follows. In § 2, we formulate the discrete contact problem. In § 3, we outline how to determine when the onset of slip or separation will occur for a contact subjected to monotonic loading. In § 4, we describe how the shakedown limit calculation can be posed as a Linear Programming optimization problem for which several solution algorithms already exist. In § 5, these calculations are applied to the example (coupled) complete contact shown in Fig. 1. Over 1000 numerical simulations of the time-evolution of the contact are then performed under a wide range of cyclic loads and initial displacement states, and the influence of initial conditions on steady state frictional energy dissipation is investigated. Finally, the implications of these results are discussed in § 6.

2. Formulation

Let us consider some two-dimensional complete contact geometry, such as that shown in Fig. 1, which has been discretized using the finite element method such that there are N nodes in potential frictional contact along the interface. We can write the reaction forces, $\boldsymbol{r}$, and the relative displacements, $\boldsymbol{u}$, at the contact nodes as

$$\boldsymbol{r} = [\boldsymbol{q}, \boldsymbol{p}]^T = [q_1, \ldots, q_N, \;\; p_1, \ldots, p_N]^T \tag{1a}$$

$$\boldsymbol{u} = [\boldsymbol{v}, \boldsymbol{w}]^T = [v_1, \ldots, v_N, \;\; w_1, \ldots, w_N]^T, \tag{1b}$$

where q_i,p_i are the shear, normal reactions and v_i,w_i are the tangential, normal relative displacements, and where $i \in \{1,\ldots,N\}$. Here we adopt the convention that p_i is positive in compression (the negative y-direction), w_i positive for a positive gap (the