European Journal of Mechanics A/Solids 49 (2015) 251–267

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Bending, buckling and free vibration analysis of size-dependent functionally graded circular/annular microplates based on the modified strain gradient elasticity theory

R. Ansari [a], R. Gholami [b,*], M. Faghih Shojaei [a], V. Mohammadi [a], S. Sahmani [a]

[a] Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran
[b] Department of Mechanical Engineering, Lahijan Branch, Islamic Azad University, P.O. Box 1616, Lahijan, Iran

ARTICLE INFO

Article history:
Received 10 January 2014
Accepted 30 July 2014
Available online 15 August 2014

Keywords:
FG Circular/annular microplates
Size-dependent mechanical characteristics
Modified strain gradient theory

ABSTRACT

A Mindlin microplate model based on the modified strain gradient elasticity theory is developed to predict axisymmetric bending, buckling, and free vibration characteristics of circular/annular microplates made of functionally graded materials (FGMs). The material properties of functionally graded (FG) microplates are assumed to vary in the thickness direction. In the present non-classical plate model, the size effects are captured through using three higher-order material constants. By using Hamilton's principle, the higher-order equations of motion and related boundary conditions are derived. Afterward, the generalized differential quadrature (GDQ) method is employed to discretize the governing differential equations along with various types of edge supports. Selected numerical results are given to indicate the influences of dimensionless length scale parameter, material index and radius-to-thickness ratio on the deflection, critical buckling load and natural frequency of FG circular/annular microplates.

1. Introduction

The use of structures that are made of functionally graded materials (FGMs) is increasing due to the smooth variation of mechanical properties along some preferred direction which leads to continuous stress distribution in these structures. Recently, FGMs have been concerned for their applications in micro-structures such as micro-electro-mechanical systems (MEMS) and atomic force microscopes (Witvrouw and Mehta, 2005; Hasanyan et al., 2008; Fu et al., 2004; Lee et al., 2006) to achieve high sensitivity and desired performance.

The dependency of deformation behavior on the size effects has been experimentally observed in the micro-bending test of the microbeams (Chong and Lam, 1999; Fleck et al., 1994; Stolken and Evans, 1998). Therefore, it is essential to consider small scale effects in the analysis of the behavior of functionally graded (FG) microbeams. Conventional continuum mechanics fails to predict the size-dependent response of the structures at micro- and nano-scale due to lacking intrinsic length scales. In recent years, several higher-order elasticity theories have been introduced to develop size-dependent continuum models (Ansari et al., 2010, 2011a, 2011b; Aydogdu and Taskin, 2007; Ansari et al., 2011c, 2012a).

By reformulating and extending the Mindlin's theory, Fleck and Hutchinson (1993) developed new type of continuum theory namely as strain gradient theory in which the second-order deformation tensor separated into the stretch gradient tensor and rotation gradient tensor which leads to additional higher-order stress components compared to the couple stress theory. After that, Lam et al. (2003) introduced modified strain gradient theory (MSGT) with three material length scale parameters relevant to dilatation gradient, deviatoric gradient and symmetric rotation gradient tensors. Several size-dependent beam and plate models have been developed based on the MSGT to capture the size effects in the micro-scale structures (Kong et al., 2009; Wang et al., 2010; Kahrobaiyan et al., 2011; Zhao et al., 2012; Ansari et al., 2012b; Kahrobaiyan et al., 2012; Ansari et al., 2012c, 2012d; Wang et al., 2011a; Wang et al., 2011b). For example, Kong et al. (2009) investigated the static and dynamic responses of Euler–Bernoulli microbeams using MSGT. They studied the effect of thickness to the material length scale parameter ratio of the micro-beams on their static deformation and vibrational behavior. Wang et al. (2010) presented Timoshenko microbeams formulations based on the MSGT.

* Corresponding author. Tel./fax: +98 1412222906.
E-mail address: gholami_r@liau.ac.ir (R. Gholami).

http://dx.doi.org/10.1016/j.euromechsol.2014.07.014
0997-7538/

Another type of the higher-order continuum theories is the couple stress theory elaborated by Mindlin and Tiersten (1962) and Koiter (1964) in which four material length scale parameters (two classical and two additional) are used to incorporate micro-structure related size effect. Various researches have been carried out in which size-dependent continuum models are developed based on couple stress theory (Eringen and Suhubi, 1964a, 1964b; Mindlin, 1964, 1965; Toupin, 1964; Mindlin and Eshel, 1968; Eringen, 1983; Vardoulaksi et al., 1998). Yang et al. (2002) first proposed the modified couple stress theory (MCST) in which the constitutive equations contain only one additional material length scale parameter which causes to create symmetric couple stress tensor and to use it more easily. This property has attracted some researchers to derive the size-dependent governing equations and corresponding boundary conditions for the microbeams and microplates (Asghari et al., 2011, 2010; Salamat-talab et al., 2012; Rahaeifard et al., 2012; Ke et al., 2012a; Thai and Choi, 2013; Thai and Kim, 2013; Ma et al., 2011). For instance, utilizing the MCST, Asghari et al. (2011, 2010) proposed the size-dependent beam models based on the Timoshenko and Euler-Bernoulli theories and investigated the static and vibration behavior of FG microbeams. Furthermore, Ke et al. (Salamat-talab et al., 2012) developed a Mindlin microplate model based on the MCST for the free vibration analysis of microplates.

In the present work, the bending, buckling and free vibration responses of FG circular/annular microplates are studied based on the modified strain gradient elasticity theory and Mindlin plate theory. The developed non-classical Mindlin plate model contains three material length scale parameters which has the capability to interpret the size effects. To analyze the bending, buckling, and free vibration characteristics of FG microplates, the generalized differential quadrature (GDQ) method is utilized to discretize the governing differential equations along with different boundary conditions. The influences of material index, dimensionless length scale parameter and radius-to-thickness ratio on the deflection, critical axial buckling loads and natural frequencies of FG circular/annular microplates are discussed in detail. Furthermore, a comparison is made between the various plate models on the basis of the classical theory (CT), MCST and MSGT.

2. Formulation of size-dependent equations of motion and corresponding boundary conditions

As it can be seen in Fig. 1, an annular microplate composed of functionally graded materials through the thickness with the inner radius a, outer radius b and thickness h is considered.

2.1. Functionally graded materials

The FG microplate is supposed to be made of ceramic and metal in a way that the materials at bottom surface ($z=-h/2$) and top surface ($z=h/2$) are metal-rich and ceramic-rich, respectively. The effective Young's modulus(E), Poisson's ratio(ν) and density(ρ) of the FG microplate can be defined as

$$E(z)=(E_c-E_m)V_f(z)+E_m, \tag{1a}$$

$$\nu(z)=(\nu_c-\nu_m)V_f(z)+\nu_m, \tag{1b}$$

$$\rho(z)=(\rho_c-\rho_m)V_f(z)+\rho_m. \tag{1c}$$

the subscripts c and m are ceramic and metal phases, respectively. By defining k as the power-low index, the volume fraction of the constituents $V_f(z)$ can be defined by a simple power low function as follows (Fares et al., 2009)

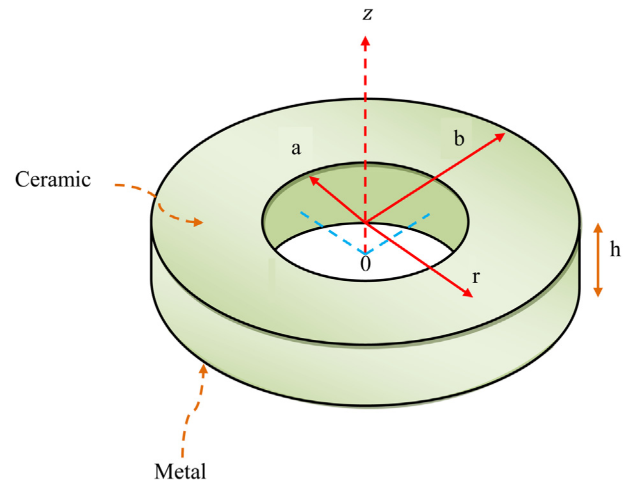

Fig. 1. Schematic diagram of an FG annular microplate: kinematic parameters, coordinate system, and geometry.

$$V_f(z)=\left(\frac{1}{2}+\frac{z}{h}\right)^k. \tag{2}$$

2.2. Modified strain gradient theory

The governing equations of motion and corresponding boundary conditions are obtained based on the first-order shear deformation plate theory and MSGT by implementing the Hamilton's principle. The principle can be presented in analytical form as

$$\delta\int_{t_1}^{t_2}\left(\Pi_T-\Pi_S+W^{\mathrm{ext}}\right)\mathrm{d}t=0, \tag{3}$$

where Π_T, Π_S and W^{ext} denote the kinetic energy, the total strain energy and the work done by external forces, respectively.

Based on the MSGT presented by Lam et al. (2003), the strain energy in a continuum made of a linear elastic material occupying region V undergoing infinitesimal deformations is stated as

$$\Pi_S=\frac{1}{2}\int_V\left(\sigma_{ij}\varepsilon_{ij}+p_i\gamma_i+\tau_{ijk}^{(1)}\eta_{ijk}^{(1)}+m_{ij}^s\chi_{ij}^s\right)\mathrm{d}V \tag{4}$$

where the components of the strain tensor ε_{ij}, dilatation gradient tensor γ_i, deviatoric stretch gradient tensor $\eta_{ijk}^{(1)}$ and symmetric rotation gradient tensor χ_{ij}^s are defined by Lam et al. (2003)

$$\begin{aligned}
&\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right),\\
&\gamma_i=\varepsilon_{mm,i},\\
&\eta_{ijk}^{(1)}=\eta_{jik}^{(1)}=\eta_{ijk}^s-\frac{1}{5}\left(\delta_{ij}\eta_{mmk}^s+\delta_{jk}\eta_{mmi}^s+\delta_{ki}\eta_{mmj}^s\right);\\
&\eta_{ijk}^s=\frac{1}{3}\left(\varepsilon_{jk,i}+\varepsilon_{ki,j}+\varepsilon_{ij,k}\right);\\
&\chi_{ij}^s=\frac{1}{2}\left(\Theta_{i,j}+\Theta_{j,i}\right),\ \Theta_i=\frac{1}{2}(\mathrm{curl}(\mathbf{u}))_{,i}.
\end{aligned} \tag{5}$$

here u_i denotes the components of the displacement vector $\boldsymbol{u}$, Θ_i expresses the infinitesimal rotation vector $\boldsymbol{\Theta}$ and the symbol of δ represents the Kronecker delta.