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On the contact stiffness and nonlinear vibration of an elastic body with a rough surface in contact with a rigid flat surface

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ABSTRACT

The surface topography of engineering rough surfaces has an enormous influence on contact mechanics of the interface and further the dynamics of the contact system. In this paper, the force-deflection characteristic and the nonlinear vibration of a rough surface interacting with a rigid flat surface are studied. A three-dimensional rough surface is constructed using a modified two-variable Weierstrass–Mandelbrot fractal function and the force-deflection is determined to be a positive power-law function. The power has the values larger than unity and increases with a rougher surface topography. Approximation of the force-deflection characteristic is also presented. Accordingly, the natural frequency is determined both exactly and approximately from the numerical calculation of the natural period and using the multiple scales method on the approximate equation, respectively. The primary resonance responses under harmonic excitation are also determined as well as the jump-up and jump-down responses. The nonlinear contact stiffness characteristics and the effect on natural frequency and the frequency responses are illustrated for different rough surface topographies. It is shown that the variation of natural frequency with amplitude and the multi-valued region of frequency responses increase with a rougher surface topography; however, the jump-up and jump-down frequencies both decrease, as well as the peak amplitude.

1. Introduction

The dynamic motions at contact interfaces are fundamental to the wear, fatigue and energy dissipation of mechanisms and mechanical devices. Some examples of such mechanical systems include rolling element bearings, gear meshes and railway wheel-rail contact (Liew and Lim, 2005; Ma, 2005; Wu and Thompson, 2006). Hertzian contact theory has been employed generally to model the contact mechanics at the interfaces. For example, Nayak (1972) studied the dynamic behaviour of Hertzian contact vibrations based on a single degree-of-freedom (SDOF) dynamic model using the first-order Harmonic Balance Method (HBM). Hess and Soom (1991) analysed the average friction reduction due to the dynamic motion using the Method of Multiple Scales (MMS) (Nayfeh and Mook, 1979). Sabot et al. (1998) analytically studied the undamped free nonlinear vibrations of a sphere-plane contact model by combining the method of power series expansion of the nonlinearity, first described by Hayashi (1964), and the MMS. Rigaud and Perret-Liaudet (2003), and Perret-Liaudet and Rigaud (2003) focused their investigations on the vibro-impact problem by using the shooting method in conjunction with parametric continuation technique. Ma et al. (2007) studied the same sphere-plane contact model with possible contact loss using a multi-term HBM (Ma and Kahraman, 2005).

For the elastic Hertzian contact interaction conducted in previous studies, for example Nayak (1972), Hess and Soom (1991), Sabot et al. (1998), Rigaud and Perret-Liaudet (2003), the force-deflection characteristic is described by a power-law relationship with the power value equal to 3/2, which corresponds to an elastic sphere on a flat rigid surface. However, in reality all engineering surfaces are rough to some degree. The actual topographies of interacting

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surfaces can have a significant influence on contact mechanics and the stiffness of the interface (Majumdar and Bhushan, 1990; Yan and Komvopoulos, 1998; Hyun et al., 2004). It is well-known that the static force-displacement characteristic dominates the dynamic performance of a system. For example, the sign of the cubic term in the Duffing equation determines whether the system is softening or hardening (Nayfeh and Mook, 1979; Brennan et al., 2008); the addition of a quadratic term to the nonlinear restoring force in the Duffing equation introduces asymmetry to the system, which causes softening behaviour (Nayfeh and Mook, 1979; Kovacic et al., 2008). Therefore, the contact force-deflection characteristic between these rough surfaces is very important to the dynamic behaviour of the system.

The aim of this paper is to determine the force-deflection characteristic of an elastic body with a rough surface in contact with a rigid flat surface and to study its effect on the free and forced vibration responses of the system. A three-dimensional rough surface is constructed using a modified two-variable Weierstrass–Mandelbrot fractal function (Yan and Komvopoulos, 1998) and the force-deflection characteristic of the rough surface on a rigid flat surface is determined using the finite element analysis (FEA). Conventionally, the surface roughness is characterized by statistical parameters of the surface height distribution function, such as standard deviations of the surface height, slope and curvature (Greenwood and Williamson, 1966; Nayak, 1973; Jackson and Green, 2006). However, the values of these statistical parameters are scale-dependent, i.e. they strongly depend on the sample size and the resolution of the measuring instrument. Fractal geometry, pioneered by Mandelbrot (Mandelbrot, 1983), has been introduced in the field of contact mechanics to perform scale-independent analysis and is characterized by continuity, non-differentiability, and self-affinity. These mathematical properties are satisfied by the Weierstrass–Mandelbrot function (Yan and Komvopoulos, 1998). The finite element method, which is possible to develop a surface model that accounts for the actual surface topographies and is generally suitable for implementation in FE software, has been widely used to study the contact of elastic solid with self-affine fractal surface (Komvopoulos and Ye, 2002; Sellgren et al., 2003; Hyun et al., 2004).

The approximation of the nonlinear force-deflection characteristic, using a third order Taylor series expansion, is also presented to derive an analytical solution. Accordingly, the natural frequency is determined both exactly and approximately. The exact natural frequency is determined from the numerical integration of an expression to give the natural period. The approximate values are determined using the MMS on the approximate equation of motion. The primary resonance responses under harmonic excitation are also determined as well as the jump-up and jump-down responses. The nonlinear contact stiffness characteristics and the effect on natural frequency and frequency responses are illustrated by choosing different power law values corresponding to different rough surface topographies.

2. Description of the dynamic model

An elastic body, which has a rough surface and mass m, is considered to be in contact with a rigid, flat surface as shown in Fig. 1(a). Deflection occurs at multiple asperities on the rough surface resulting in a contact stiffness k. A simple dynamic model for the interaction between the mass and the contact stiffness is shown in Fig. 1(b). It can be seen that this is a single degree-of-freedom (SDOF) system, which has been assumed for simplicity and is considered to be valid for low frequency behaviour (Sabot et al., 1998; Rigaud and Perret-Liaudet, 2003). The static deflection z_s is due to the weight of body.

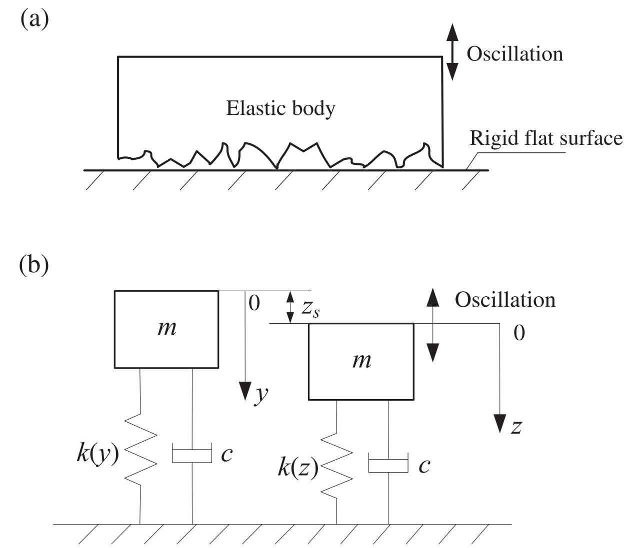

Fig. 1. (a) The model for an elastic body with rough surface in contact with a rigid flat surface, (b) the single degree-of-freedom (SDOF) model representation of Fig. 1(a).

The SDOF nonlinear stiffness system in Fig. 1(b) has been used extensively to study the dynamics of a sphere in contact with a rigid surface, for example Hess and Soom (1991), Sabot et al. (1998), Rigaud and Perret-Liaudet (2003), Xiao et al. (2011). In these studies, the static force-deflection characteristic was taken to be $f = k_0 y^{3/2}$, where f is the normal load, the constant k_0 depends on the material properties and the geometry of the contact area, and y is the deflection of the body due to elastic compression in the contact region with $y = z + z_s$, and linear viscous damping was assumed (Sabot et al., 1998; Ma et al., 2007). For a more general nonlinear contact stiffness model with a restoring force given by $f(y)$, the equation of motion about the static equilibrium position for harmonic excitation of the model in Fig. 1(b) is given by.

$$m\ddot{z} + c\dot{z} + f(z + z_s) - mg = F\cos(\omega t) \tag{1}$$

Note that Eq. (1) is only valid when the elastic body is in contact with the rigid flat surface, i.e. when $z \geq -z_s$. Introducing the following non-dimensional variables for displacement, time, excitation frequency and damping respectively.

$$u = \frac{z}{z_s}, \quad \tau = \omega_s t, \quad \Omega = \frac{\omega}{\omega_s} \quad \text{and} \quad \zeta = \frac{c}{2m\omega_s},$$

where $\omega_s = \sqrt{k(z_s)/m}$ is the undamped natural frequency at the static equilibrium position, the subsequent dimensionless equation of motion is given by

$$u'' + 2\zeta u' + \frac{f(uz_s + z_s) - mg}{k(z_s)z_s} = \frac{F}{k(z_s)z_s}\cos(\Omega\tau), \quad u \geq -1 \tag{2}$$

in which $k(z_s)$ is the stiffness at the static equilibrium position z_s and the $'$ denotes differentiation with respect to non-dimensional time τ.

3. Force-deflection characteristic of the rough surface on a rigid flat surface

To determine the nonlinear contact stiffness $k(z)$ between the elastic body and the rigid surface shown in Fig. 1(b), a three-dimensional rough surface is constructed, and then the force-deflection characteristic is computed. The three-dimensional