European Journal of Mechanics A/Solids 49 (2015) 345–361

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A two-scale time-dependent model of damage: Influence of micro-cracks friction

Aleksandra Wrzesniak [a], Cristian Dascalu [b, c, *], Pierre Bésuelle [a]

[a] Grenoble-INP, UJF-Grenoble 1, CNRS UMR 5521, Lab. 3SR, 38041 Grenoble, France
[b] UPMC Univ. Paris 06, CNRS UMR 7190, Institut Jean-Le-Rond-d'Alembert, 75005 Paris, France
[c] CNRS, UMR 7190, Institut Jean-Le Rond-d'Alembert, 75005 Paris, France

ARTICLE INFO

Article history:
Received 16 January 2014
Accepted 21 August 2014
Available online 2 September 2014

Keywords:
Frictional microcracks
Asymptotic homogenization
Time-dependent damage

ABSTRACT

The aim of this paper is to present a micro-mechanical damage model for quasi-brittle materials that accounts for friction effects on microcracks. We use homogenization based on asymptotic developments to deduce the overall damage behavior starting from explicit descriptions of elementary volumes with micro-cracks. A time-dependent propagation criterion is assumed for the evolution of cracks at the small scale. An appropriate micro-mechanical energy analysis is proposed leading to a damage evolution law that accounts for friction effects, strain rate dependency, stiffness degradation, material softening and size effects. Numerical results are presented in order to illustrate the local and structural effective damage response. Mesh-independency is proved for the finite-element solutions, as a consequence of the regularizing effect of time.

1. Introduction

The accurate modeling of the inelastic behavior and failure of quasi-brittle materials is necessary for many engineering applications. Their heterogeneous nature has a significant influence on the observed macroscopic behavior. Various phenomena occurring at the level of the macrostructure, such as inelastic non-linear stress–strain response, degradation of the elastic properties, material softening and induced anisotropy, originate from the physics and mechanics of the underlying microstructure. At the micro-scale, the materials contain various sources of heterogeneities such as cracks, pores, inclusions or grain boundaries. Nucleation and growth of micro-cracks are typical damage mechanisms in brittle and quasi-brittle solids like rocks, concrete or ceramics. Due to their heterogeneous microstructure and dissipative phenomena like micro-cracks growth and frictional sliding on crack faces, the macroscopic behavior includes inelastic non-linear stress–strain response, degradation of elastic properties, induced anisotropy and unilateral effects related to cracks closure. Studying the relation between micro-structural phenomena and the macroscopic response allows to predict the behavior of existing materials and make engineering design more efficient.

Since the first introduction of the scalar damage concept by Kachanov (1958) and Rabotnov (1963) for creep of metals, continuum damage mechanics has become an emerging field of active research. From the point of view of the construction procedure, one can classify the damage models as phenomenological or micro-mechanical. While the phenomenological models do not necessarily take into account the aspects of the small scale, the models in the second class generally involve upscaling from microstructural aspects.

In the recent years, considerable efforts have been made to establish a link between micro-structural fracture phenomena and the corresponding macroscopic behaviors. Many researchers focused on the development of macroscopic modeling starting from considerations of micro-mechanical analysis of a multi-cracked medium through homogenization procedures. In this way, the micro-mechanical arguments may lead to a better understanding of the phenomena observed at the macroscopic level in the experiments.

Among various models found in the literature, one can mention works dealing with frictional sliding micro-cracks, like for instance Halm and Dragon (1998), Lawn and Marshall (1998), Poon et al. (2011). Various micro-mechanical studies have been proposed in order to model evolving frictional micro-cracks like in Kachanov (1982). This approach was further developed by many authors,

* Corresponding author. Current address: Laboratoire d'Etude des Microstructures et de Mécanique des Matériaux, Université de Lorraine, CNRS UMR 7239, Ile du Saulcy, F-57045 Metz, France

E-mail addresses: christian.dascalu@gmail.com, cristian.dascalu@univ-lorraine.fr (C. Dascalu).

http://dx.doi.org/10.1016/j.euromechsol.2014.08.006
0997-7538/

such as Nemat-Nasser and Obata (1988), Gambarotta and Lagomarsino (1993) for noninteracting micro-cracks. For interacting and growing penny-shaped micro-cracks in the context of the self-consistent method we can refer to Lee and Ju (1991).

More general approaches, phenomenological or micro-mechanically inspired, coupling damage and plasticity can be found, for instance, in Gambarotta (2004), Tashman et al. (2005), Xie et al. (2011), Kruch and Chaboche (2011), Abu Al-Rub and Darabi (2012) and Shojaeia et al. (2013).

An interesting approach is proposed by Andrieux et al. (1986). An important assumption made by these authors is that for the cell problem the crack is rectilinear and small in comparison with dimensions of the basic cell. In this case an infinite medium approximation is allowable. A three-dimensional strain-based micro-mechanical analysis (based on Andrieux et al. (1986)) of the brittle damage incorporating micro-crack closure and friction can be found in the work of Pensée et al. (2002). This model shows a capability to predict several important aspects of brittle anisotropic damage (oriented mesocrack growth, unilateral effects) and hysteretic behavior due to friction phenomena.

A micro-mechanical modeling of closed frictional cracks as flat ellipsoidal inhomogeneities is proposed by Barthélémy et al. (2003). Three interface friction laws were considered in this paper: von Mises, Coulomb and Drucker-Prager. The macroscopic behavior of such a cracked medium was obtained by means of homogenization technique based on Eshelby's results.

The behavior of closed frictional cracks is also studied by Zhu et al. (2007) in context of Eshelby solution-based homogenization procedure. They consider a family of closed frictional microcracks obeying to the classical interface Coulomb law. In this paper different homogenization schemes are used and the damage evolution law was determined using the standard thermodynamics framework.

In the context of asymptotic homogenization approach, Leguillon and Sanchez-Palencia (1982) performed the homogenization of a two-dimensional linear elastic body with cracks with friction. Telega (1990) proposed a model, which deals with the homogenization of a three-dimensional, hyper-elastic solid weakened by periodically distributed microcracks in presence of the Signiorini's condition and friction. Both these studies concern physical situations in which the micro cracks do not evolve.

A new procedure for homogenization leading to damage evolution laws was proposed in our recent contributions (Dascalu et al., 2007, 2010; Dascalu, 2009; Keita et al., 2014; François and Dascalu, 2010; Markenscoff and Dascalu, 2012). The micro-mechanical damage models were obtained by using an upscaling procedure, which combines a periodic homogenization based on asymptotic developments (Bakhvalov and Panasenko, 1989) and micro-fracture energy analysis. The damage evolution laws are completely deduced from micro-structural analysis by homogenization, without phenomenological assumptions at the macroscopic scale. In the resulting damage law the normalized micro-crack length appears as a damage variable and the cell size as a material length parameter. In this way, the resulting damage law naturally accounts for size-effects.

In the present work, we extend the previous results in order to include friction effects on micro-cracks. We construct a time-dependent damage model, deduced by homogenization for micro-cracks following a subcritical criterion of propagation. Leguillon and Sanchez-Palencia (1982) or Telega (1990) already considered friction effects for stationary cracks. On the other side there are other contributions for evolving frictional micro-cracks, which use different homogenization techniques like for instance Andrieux et al. (1986), Halm and Dragon (1998), Pensée et al. (2002), Barthélémy et al. (2003) and Zhu et al. (2007).

For complex micro-structural fracture evolutions, computational homogenization schemes may appear more appropriate (Geers et al., 2010; Bilbie et al., 2008). However, the aim of the present approach is different since we want to obtain analytical forms of the damage laws. Such a model, even if deduced under more restrictive assumptions, illustrate the way in which damage laws account for friction effects and may serve as basis for further developments.

The paper is organized as follows. First, the model problem is presented, including the asymptotic homogenization procedure and the macroscopic equilibrium equations. Then, we perform the energy analysis leading to the macroscopic damage law and we deduce the time-dependent damage law. The next sections present numerical results concerning the homogenized coefficients and macroscopic parameters related with the dissipation of the energy due to frictional sliding of the micro-cracks. Finally, the model is studied by means of numerical examples at macroscopic local and structural level, emphasizing the influence of microscopic friction on the time-dependent response.

2. The initial problem

We consider a 2D isotropic elastic medium containing a large number of micro-cracks with a locally periodic distribution. We denote by 135° the coordinate system aligned to the directions of periodicity, as represented in Fig. 1. Let $\mathcal{B}$ be a whole body, a bounded domain of $\mathcal{R}^2$ containing $\mathcal{N}$ micro-cracks $\mathcal{C}_n$, $n = 1,\ldots,\mathcal{N}$ and let's denote by $\mathcal{B}_{\tilde{s}} = \mathcal{B}/\mathcal{C}$ its solid part, where $\mathcal{C} = \cup_{n=1}^{N}\mathcal{C}_n$.

In the solid part $\mathcal{B}_{\tilde{s}}$, we have the equilibrium equations

$$\frac{\partial \sigma^{\varepsilon}_{ij}}{\partial x_j} = 0 \tag{2.1}$$

and the linear elasticity constitutive relations

$$\sigma^{\varepsilon}_{ij} = a_{ijkl} e_{xkl}(\mathbf{u}^{\varepsilon}) \tag{2.2}$$

where $\mathbf{u}^{\varepsilon}$ and $\boldsymbol{\sigma}^{\varepsilon}$ are the displacement and the stress fields. We denoted by $e_{xij}(\mathbf{u}) = ½(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ the small strain tensor with respect to x variables.

Concerning the boundary conditions on the faces of the micro-cracks, we may have traction free opening

$$\boldsymbol{\sigma}^{\varepsilon}\mathbf{N} = 0 \tag{2.3}$$

or frictional contact

$$[\boldsymbol{\sigma}^{\varepsilon}\mathbf{N}] = 0; \quad \mathbf{N}\boldsymbol{\sigma}^{\varepsilon}\mathbf{N} < 0; \quad [\mathbf{u}^{\varepsilon}\cdot\mathbf{N}] = 0 \tag{2.4}$$

with

$$|\mathbf{T}\boldsymbol{\sigma}^{\varepsilon}\mathbf{N}| < -\mu_f \mathbf{N}\boldsymbol{\sigma}^{\varepsilon}\mathbf{N} \tag{2.5}$$

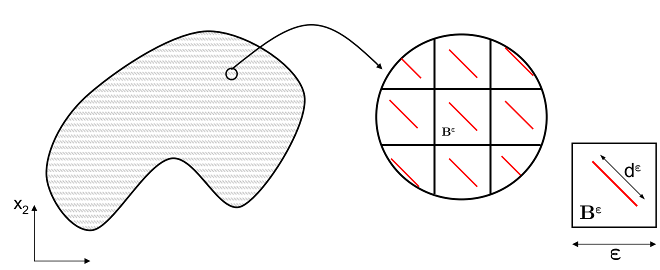

Fig. 1. Fissured medium with locally periodic microstructure.