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Closed form solutions for predicting static and dynamic buckling behaviors of a drillstring in a horizontal well

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ABSTRACT

The nonlinear static and dynamic buckling (snaking motion) analysis of a rotating drilling string laterally constrained in a horizontal well is presented, through a group of fourth-order nonlinear partial differential equations. The analytical approximate solutions to the static and dynamic buckling are obtained via combining Newton linearization with Harmonic Balance Method, and Galerkin's method, respectively. On the basis of the analytical approximate solutions, static post-buckling deformation, critical dynamic buckling load, and two different kinds of snaking motions (i.e. the pipe moves up and down around its static buckling configuration; the pipe moves from one side of the wellbore to the other side) are investigated. Accuracy of the approximate solutions is verified by comparing with numerical solutions. Theoretical results are useful for practical design applications related to calculation of buckling loads and selection of bottom hole-assembly (BHA) elements and pipe rotational speeds. What's more, the solving procedures of accurate analytical approximate solutions to the snaking motions yield rapid convergence with respect to exact numeric solutions. The present results are valid for small as well as large oscillation amplitudes.

1. Introduction

The success or failure of drilling engineering (especially, in ultra-deep drilling of oil/gas and deep continental scientific drilling exploration) is generally determined by the work effect of drillstring, namely, whether drillstring could safely work (Wicks et al., 2008). The reason is that the drillstring is used to not only transfer torque and motion from the rig to the bit, but also to convey drilling fluid. The force condition of the drillstring, however, is very difficult: the combined action of axial and lateral loads, torque, pressure, the contact force from wellbore and frictional and viscous drags, which could lead to harmful vibrations. The drillstring, therefore, may lose its stability and develop very complicated snaking and whirling motions, under some critical conditions, and even collapse to result in drilling accidents. Thus, it is important and meaningful to investigate the dynamic and buckling behavior of drillstring for the science and technologies in petroleum engineering, deep continental scientific drilling and other related fields (Gulyayev et al., 2009; Tan and Gan, 2009; Gao and Miska, 2009, 2010).

Especially, it is crucially important for successful drilling to understand the mechanisms of both dynamic stability and post-buckling behavior of rotating drilling pipe in a horizontal well. Static stability and post-buckling behaviors of pipe in vertical, inclined, horizontal, and curved wellbores are first studied by Paslay and Bogy (1964). Based on the principle of minimum potential energy, the problem of helical buckling of a vertical tube was first analyzed by Lubinski et al. (1962). Since then, Dawson and Paslay (1984), Cheatham and Pattillo (1984), Mitchell (1988), He and Kyllingstad (1993), Miska and Cunha (1995), Huang and Pattillo (2000), Gao et al. (2002), Yuan and Wang (2012) have studied helical buckling of tubes in vertical, horizontal or inclined wellbores, based on the energy method, finite element method and the discrete singular convolution, respectively. Experimental study of helical buckling of a horizontal rod in a tube was performed by McCann and Suryanarayana (1994). While Wicks et al. (2008) reviewed available analytical and experimental results on the structural behavior of constrained horizontal cylinders subjected to axial compression, torsion, and gravity. Gao and Miska (2009) have taken the friction effects into consideration in the theoretical analysis. By using the discrete singular convolution (DSC)

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algorithm, Wang and Yuan (2012) investigate the effects of friction and boundary constraints on the nonlinear buckling behavior of a relatively short rod constrained in a rigid horizontal cylinder and subjected to axial compression, gravitational and frictional loads.

In spite of many papers discussing the dynamic vibrations of pipe, including axial, lateral, and torsional vibration (Besaisow and Payne, 1988; Jansen, 1992; Ashley et al., 2001; Samuel et al., 2006), however, the papers focusing on both dynamic stability and buckling behavior of rotating pipe constrained in a horizontal or inclined well are very few. Neglecting the nonlinear interactions between axial vibration and lateral buckling, the dynamic stability of drilling pipe under fluctuating weight on bit are investigated by Dunayevsky et al. (1993), and Heisig and Neubert (2000) proposed a theoretical model to predict the influence of revolutions per minute on the dynamic stability and lateral vibration of drilling strings in horizontal wells. Assuming a viscous damping for both the axial and torsional motions, linear stability analysis of a state dependent delayed, coupled two DOF model of drill-string vibration is investigated by Nandakumar and Wiercigroch (2013). Actually, the dynamic behavior of the drillstring is nonlinear. Therefore, it is difficult for both models of linear differential equations to accurately describe dynamic stability of drilling pipe. Recently, Gao and Miska (2010) proposed a dynamic buckling model to discuss two different patterns of snaking motion of drill pipe. The elliptic function solutions of this model was obtained, which show that the rotating speed will not affect the critical load of snaking motion. Compared with the experimental observations and measurements of dynamic motion of rotating pipe in a horizontal well, the validity of theoretical results is illustrated. Considering the case when both ends of the rod are at the center of the tube cross section, Li and Chen (2014) use elastica model to calculate the deformation of a clamped–clamped rod under end twist and constrained inside a straight tube. The results show that the small-deformation theory is inadequate in representing the constrained rod in a tube with radius-to-length ratio of 0.05. In fact, the finite element method and transient numerical integration could be directly used to solve dynamic problem of the drill string (Heisig and Neubert, 2000; Sampaio et al., 2007; Filiz and Ozdoganlar, 2010; Ritto et al., 2013; Kong et al., 2013). However, compared to the direct numerical solutions, the analytical approximate ones could show their advantage of describing the depending relationship of the response with the geometric and physical parameters of the system, for their explicit expressions.

In this paper, the nonlinear buckling and snaking motion analysis of a rotating drilling string laterally constrained in a horizontal well is discussed. Firstly, the analytical approximate solutions of the static buckling of the drillstring are obtained via combining the Chebyshev polynomials, Newton linearization and Harmonic Balance Method. Secondly, the two different patterns of snaking motion are discussed by employing Improved Harmonic Balance Method and Galerkin's method, respectively. Finally, accuracy of the approximate solutions is verified by comparing with numerical solutions. Theoretical results are useful for practical design applications.

2. Theoretical model

For the title problem, the drillstring in continuous contact with the wall of wellbore is assumed to be slender and whose deformation is elastic, because of the small clearance between the drillstring and the wellbore. The friction in system and the effect of torsion are not treated. The spatial position of a point in the centerline of the drillstring at any time can be completely determined by its axial and angular displacements $u(x,t)$, and $\theta(x,t)$ as shown in Fig. 1. Where W_n and q are the contact force per unit

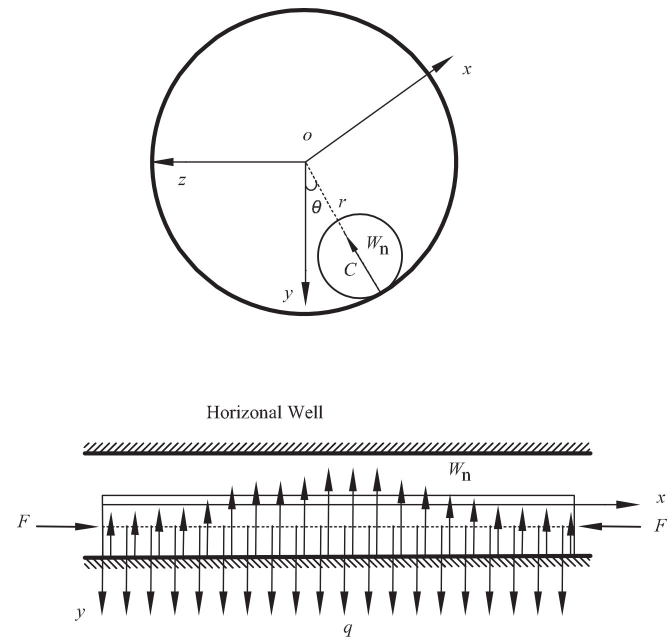

Fig. 1. Illustration of the drillstring in a horizontal wellbore.

length between drillstring and wellbore, and weight of the drillstring per unit length, respectively. In this paper, we do not pay attention to the contact force W_n, thus, the expression of which would not be presented. Through derivation and simplification, the governing equation of a drillstring within a wellbore, including the boundary conditions, could be expressed as in the form (Gao and Miska, 2009, 2010)

$$EA\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} + EAr^2\frac{\partial\theta}{\partial x}\frac{\partial^2\theta}{\partial x^2} = 0 \tag{1}$$

$$\begin{aligned} & EIr\left[\frac{\partial^4\theta}{\partial x^4} - 6\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial^2\theta}{\partial x^2}\right] + r\frac{\partial}{\partial}\left(F\frac{\partial\theta}{\partial x}\right) + J_p\omega\left(\frac{\partial\omega_r}{\partial x} - \omega_\theta\frac{\partial\theta}{\partial x}\right) \\ & + \rho Ag\sin\theta + \rho Ar\frac{\partial^2\theta}{\partial t^2} = 0 \end{aligned} \tag{2}$$

$$\begin{aligned} & \left.\frac{\partial^2\theta}{\partial x^2}\right|_{x=0} = 0,\ \theta(0,\tau) = 0 \\ & \left.\frac{\partial^2\theta}{\partial x^2}\right|_{x=L} = 0,\ \theta(L,\tau) = 0 \\ & u(0,\tau) = 0,\ u(L,\tau) = -u_L \end{aligned} \tag{3}$$

where

$$F(x,t) = -EA\frac{\partial u}{\partial x} - \frac{1}{2}EAr^2\left(\frac{\partial\theta}{\partial x}\right)^2 \tag{4}$$

$$\omega_r = -r\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial t},\quad \omega_\theta = r\frac{\partial^2\theta}{\partial x\partial t} \tag{5}$$

Here $x\in[0,L]$ is the axial coordinate along with the center line of the drillstring, and EI, L, A, J_p and ρ are bending rigidity, the length, cross-section area, inertial moment and the density of the drillstring, respectively. The radial clearance between the pipe and the