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# Numerical homogenization of mesoscopic loss in poroelastic media

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ABSTRACT

This contribution deals with the numerical homogenization of mesoscopic flow phenomena in fluid-saturated poroelastic media. Under compression, mesoscopic heterogeneities induce pore pressure gradients and consequently pressure diffusion of the pore fluid. Since this process takes place on a scale much smaller than the observable level, the dissipation mechanism is considered as a local phenomenon. The heterogeneous poroelastic medium is substituted by an overall homogeneous Cauchy medium accounting for viscoelastic properties. Applying volume averaging techniques we derive a consistent upscaling procedure based on an appropriate extension of the Hill-Mandel lemma. We introduce various sets of boundary conditions for the poroelastic problem and discuss the relation between size of the $\mathrm{SVE}_m$ (Statistical Volume Element) and maximum diffusion length. Numerical examples for two- and three-dimensional problems are given.

## 1. Introduction

Seismic attenuation in partially saturated porous rocks is of enormous scientific and economic interest. It has been recently observed that oil and gas reservoirs frequently exhibit high P-wave attenuation, especially at low seismic frequencies, see (Chapman et al., 2006) and references therein. Significant frequency dependence of P-wave velocities are also commonly observed in such media (Korneev et al., 2004; Goloshubin et al., 2006). Attenuation can, therefore, be used in the interpretation and inversion of seismic data for inferring pore fluid saturation or quality of reservoir rock. Ba et al. (2013) performed inversion of seismic data using an analytical solution to calculate attenuation and dispersion caused by wave-induced fluid flow in a medium containing mesoscopic spherical inclusions. High P- and S-wave attenuation can also be related to fractures and their connectivity in fluid-saturated rocks (Maultzsch et al., 2003; Rubino et al., 2013; Quintal et al., 2014). Because fractures and joints can drastically increase the effective hydraulic conductivity of porous rocks, using seismic data (microseismic, surface reflection seismic, vertical seismic profile) to detect and characterize them is of extreme importance in applied geophysics. For many applications, such as geothermal energy or hydrocarbon production from shale reservoirs, information about fracture connectivity is targeted as primary decision factor. Some analytical solutions for characterizing attenuation based on simple distributions of fractures have been proposed in recent years, for example (Chapman, 2003; Gureveich et al., 2009). However, to calculate and understand seismic attenuation for media having realistic fluid and/or fracture spatial distributions, we may perform numerical modelling in poroelastic media.

### 1.1. Rock physics background

At seismic frequencies ($1 < f < 200\ \mathrm{s}^{-1}$), wave-induced fluid flow caused by fluid pressure differences between mesoscopic heterogeneities is a major cause of P-wave attenuation in partially saturated porous rocks (White, 1975; Pride et al., 2004; Müller et al., 2010). Tisato and Quintal (Tisato and Quintal, 2013) used laboratory data to show that seismic attenuation in a partially water-saturated sandstone sample is dominantly caused by wave-induced fluid flow on a mesoscopic scale. Here, the mesoscopic scale is considered to be much larger than the pore size but much smaller than the wavelength, see Fig. 1. Partially saturated rocks are approximated on meso level by a poroelastic medium with regions fully saturated by one fluid and other regions fully saturated by another fluid. This is frequently referred to as patchy saturation. However, mesoscopic heterogeneities may also occur in solid frame material properties (Pride et al., 2004). These heterogeneities represent regions with different elastic moduli (grains, frame)

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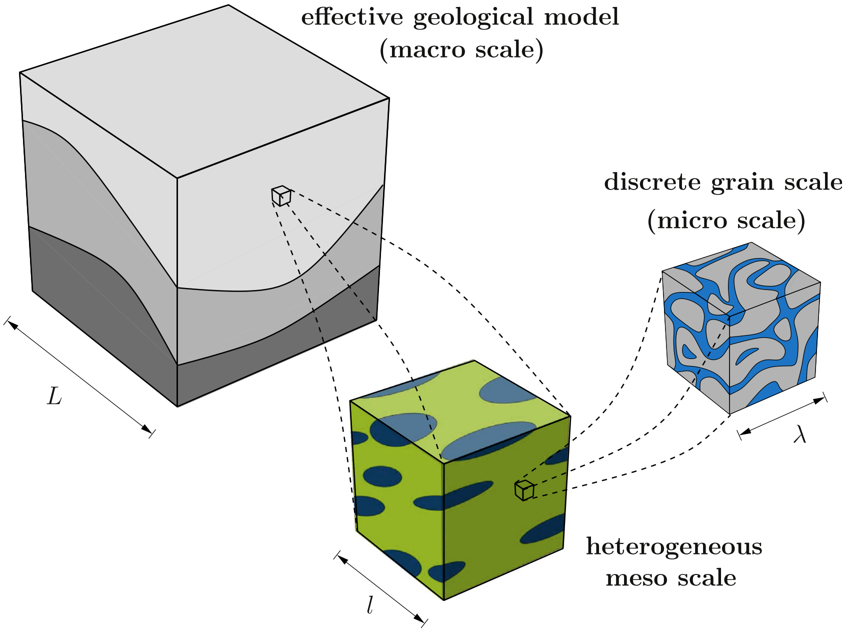

**Fig. 1.** Effective geological models comprise heterogeneities on multiple length scales. In our considerations, the smallest scale is the biphasic micro scale of discrete grains and pore channels. The heterogeneous micro level can be replaced by an effective poroelastic medium on meso level governed by Biot's equations of motion. The meso level again is assumed to exhibit spatially varying poroelastic material parameters and is substituted by a homogeneous, viscoelastic medium on macro level. All length scales are considered to be perfectly separated ($\lambda \ll l \ll L$).

within the medium. Thus, passing seismic waves lead to a reduced pore space depending on the solid frame stiffness and consequently to fluid pressure differences between regions with varying material parameters. The pressure gradients induce pressure diffusion caused by viscous fluid flow and, therefore, part of the energy involved in the wave propagation is lost due to viscous dissipation. From an overall (macroscopic) point of view, this diffusive fluid flow due to mesoscopic heterogeneities can be considered as a local phenomenon. Thus, the maximal diffusion length of the flow processes is much smaller than the inducing wave length and, therewith, much smaller than the characteristic length of the effective scale ($l \ll L$). Under these circumstances, the overall medium can be substituted by a homogeneous Cauchy continuum exhibiting viscoelastic properties, as it has been proposed in the seminal work of White et al. (White, 1975; White et al., 1975) for 1d layered media. However, White's effective viscoelasticity does not correspond to a simple 3-parameter (Maxwell-Zener) rheology, see Fig. 2. Thus, more enhanced viscoelastic models based on additional intrinsic relaxation times are required for a more precise modelling of the higher frequency behaviour. Dutta and Odé (1979a, 1979b) showed that patchy saturation can be simulated using the dynamic Biot equations (Biot, 1962) with spatially varying petrophysical parameters for the poroelastic medium. However, numerically simulating seismic wave propagation on the fully resolved model to calculate attenuation due to wave-induced fluid flow at low (seismic) frequencies is extremely expensive from the computational point of view due to the involved multiple temporal and multiple spatial scales (Carcione and Quiroga-Goode, 1995; Carcione et al., 2003, 2010). Computationally more efficient techniques have been recently proposed (Masson and Pride, 2007; Rubino et al., 2009; Wenzlau et al., 2010). These methods are based on the solution of a poroelastic initial boundary value problem on the level of a certain mesoscopic volume element considered representative for the particular heterogeneous

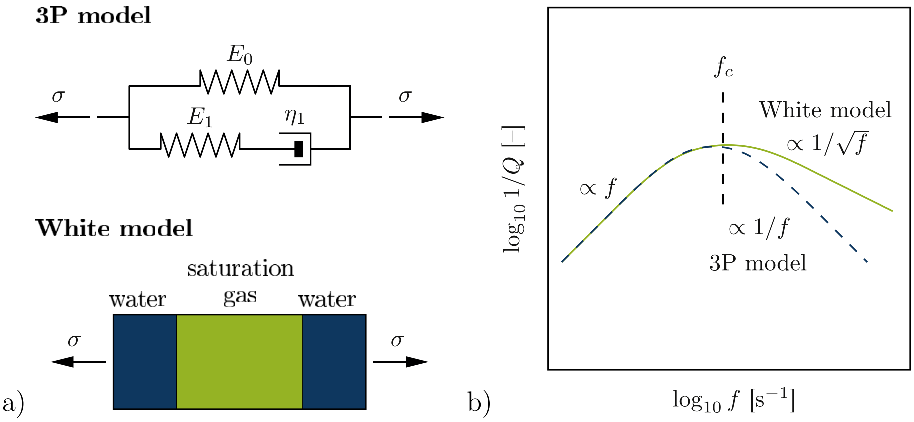

**Fig. 2.** a) Scheme of viscoelastic 3-parameter (3P) model and White model for patchy saturated poroelastic media, see (White, 1975). b) Frequency dependence of loss factor $1/Q$ for 3P and White model. For frequencies larger than transition frequency $f_c$ the characteristics a proportional to $1/f$ (3P) and $1/\sqrt{f}$ (White).