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A new domain-independent interaction integral for solving the stress intensity factors of the materials with complex thermo-mechanical interfaces

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ABSTRACT

Thanks to the feature of being domain-independent for the interfaces with discontinuous mechanical properties, the domain-independent interaction integral (DII-integral) has become a quite effective method to solve the stress intensity factors (SIFs) of the materials with complex interfaces. However, the DII-integral loses its domain-independence feature for the interface with a discontinuous thermal expansion coefficient, which causes a great difficulty in applying the DII-integral to deal with the materials with complex thermo-mechanical interfaces under thermal loading. In order to overcome this difficulty, this paper proposes a zero-mean stress auxiliary field and employs it to establish a new DII-integral. The new DII-integral is domain-independent for the interface with discontinuous mechanical properties as well as discontinuous thermal expansion coefficient, and its expression does not contain any thermal property parameters. These features greatly facilitate its practical application in solving the SIFs of the material with complex thermo-mechanical interfaces. Finally, the DII-integral combined with the extended finite element method (XFEM) are used to solve the SIFs of adiabatic cracks in order to verify its effectiveness.

1. Introduction

A number of composite materials, such as particulate, fiber-reinforced, and granular composites, contain complex interfaces that usually act as a source of defects such as cracks and holes, due to material property mismatch. Therefore, these composite materials are more prone to fracture than homogeneous materials. In particular, when a composite comprised of components that possess disparate thermal expansion coefficients is subjected to temperature changes, the composite will develop residual thermal stresses, which may act as the driving force of crack growth (Nairn, 1997). With the development of multi-scale mechanics, there has been increased interest in the correlation of a material microstructure with the development of its local strain fields, as this correlation is believed to influence the macroscale material response (Gonzalez and Lambros, 2013). As the scaling dimension decreases, the interfaces must be considered in the fracture analysis of composite materials.

A key step in analyzing crack propagation problems is to solve the crack-tip parameters effectively. Among the available methods for solving critical parameters characterizing crack-tip fields, Rice's J-integral (Rice, 1968) has been used with a great success because of its path-independence for homogeneous materials. However, the J-integral cannot distinguish between contributions due to crack opening and those due to shear in mixed-mode crack problems. In order to separate mode-I and mode-II stress intensity factors (SIFs), Chen and Shield (1977) proposed the interaction integral (I-integral) that consists of the cross terms in the J-integral under a superimposed load of two admissible states. By defining a suitable auxiliary field, Nakamura (1991) developed the I-integral to decouple the SIFs of a two-dimensional (2D) bimaterial interface crack. With a highly singular auxiliary field, researchers (Kfouri, 1986; Cho et al., 1994) demonstrated that the I-integral is also a powerful tool for extracting the T-stress, namely, the second term in William's eigenfunction expansion of the crack-tip stress field, for interior and interface cracks. As the path-independent integral attracted increasing interest, Moran and Shih (1987) determined that the numerical evaluation of contour integrals may be a source

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of inaccuracy and that the domain integrals can circumvent this potential inaccuracy. Therefore, a number of researchers (Gosz et al., 1998; Gosz and Moran, 2002; Johnson and Qu, 2007) introduced equivalent domain integrals of the I-integral for three-dimensional (3D) curved cracks. Apart from homogeneous materials, Dolbow and Gosz (2002) developed the I-integral for functionally graded materials (FGMs), a category of widely studied materials with properties varying continuously (Guo et al., 2004; Birman and Byrd, 2007; Guo and Noda, 2007; Anandakumar and Kim, 2010). Through a series of investigations on FGMs, Kim and Paulino (2003, 2005) provided three alternative definitions of the auxiliary fields for the I-integral by which to extract the SIFs and the T-stress. KC and Kim (2008) showed that these three definitions of the auxiliary fields are also effective for FGMs under thermal loading.

In practical implementation, the J-integral and the conventional I-integral must be kept away from material interfaces, which causes a difficulty in solving the SIFs of the materials with complex interfaces. This difficulty is overcome by a domain-independent I-integral (DII-integral) developed by Yu et al. (2009, 2010, 2012). The DII-integral has an attractive feature of being domain-independent for mechanical interfaces across which the mechanical properties are discontinuous. This feature makes the DII-integral one of the most promising techniques in dealing with the materials with complex interfaces. The DII-integrals have shown their superiority in solving the SIFs of cracks in isotropic bi-materials, FGMs and fiber-reinforced composites (Pathak et al., 2012; Bhattacharya et al., 2013; Cahill et al., 2014) and in orthotropic bi-materials and FGMs (Esna Ashari and Mohammadi, 2011; Bayesteh and Mohammadi, 2013; Hosseini et al., 2013). However, when studying thermal fracture problems, Guo et al. (2012) found that the I-integral lost its domain-independence feature for the interface across which the thermal expansion coefficient is discontinuous. This causes a great difficulty in thermal fracture analysis of the materials with complex thermal interfaces. In order to overcome this difficulty, the present paper first proposes a new auxiliary field to eliminate the interface integral, and thus establishes a new DII-integral for thermo-mechanical interfaces across which both thermal and mechanical properties are discontinuous. Finally, four representative crack problems subjected to thermal loading are investigated in order to verify the validity of the new DII-integral.

2. Establishment of a new DII-integral

2.1. I-integral and its equivalent domain integral

As shown in Fig. 1, Γ_ε is a contour around the crack tip and n_i is the unit outward normal vector to Γ_ε. For a 2D linear thermo-elasticity, the J-integral is defined as (Rice, 1968)

$$J = \lim_{\Gamma_\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left(\frac{1}{2} \sigma_{jk} \varepsilon_{jk}^{m} \delta_{1i} - \sigma_{ij} u_{j,1} \right) n_i \mathrm{d}\Gamma \tag{1}$$

Here, u_j is the displacement, σ_{ij} is the stress, $\varepsilon_{ij}^{m} = \varepsilon_{ij} - \varepsilon_{ij}^{\mathrm{th}}$ is the mechanical strain, ε_{ij} is the total strain, $\varepsilon_{ij}^{\mathrm{th}}$ is the thermal strain and

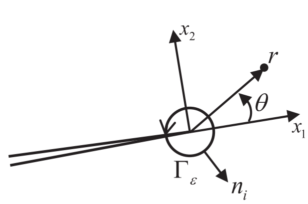

Fig. 1. Schematic diagram of a crack tip contour.

δ_{ij} is the Kronecker delta. Throughout, the repetition of an index in a term denotes a summation with respect to that index over its range from 1 to 2, and a comma denotes a partial derivative.

Superimposing the actual state (u_i) and an auxiliary state (u_i^{aux}) leads to another equilibrium state for which the J-integral $J^{(S)}$ can be divided into three components of the actual J-integral, the auxiliary J-integral and the I-integral, i.e., $J^{(S)} = J + J^{\mathrm{aux}} + I$. Eliminating J and J^{aux} from $J^{(S)}$, one can obtain the I-integral as

$$I = \lim_{\Gamma_\varepsilon \to 0} \int_{\Gamma_\varepsilon} P_{1i} n_i \mathrm{d}\Gamma \tag{2}$$

where

$$P_{1i} = \frac{1}{2} \left(\sigma_{jk}^{\mathrm{aux}} \varepsilon_{jk}^{m} + \sigma_{jk} \varepsilon_{jk}^{\mathrm{aux}} \right) \delta_{1i} - \sigma_{ij}^{\mathrm{aux}} u_{j,1} - \sigma_{ij} u_{j,1}^{\mathrm{aux}} \tag{3}$$

is the mutual energy momentum tensor in the spirit of Eshelby's concept.

Generally, the infinitesimal contour integral is generally converted into an equivalent domain integral in practical computations. As shown in Fig. 2, consider an integral domain A cut by an interface Γ_I into two sub-domains A_1 and A_2. In each sub-domain, the thermo-mechanical properties are assumed to be continuously differentiable. By introducing a weight function q with the value varying from 1 on Γ_ε to 0 on $\Gamma_B = \Gamma_{B1} + \Gamma_{B2} + \Gamma_{B3}$, the I-integral can be expressed as:

$$I = -\lim_{\Gamma_\varepsilon \to 0} \oint_{\Gamma_1^{\mathrm{O}}} P_{1i} n_i q \mathrm{d}\Gamma - \oint_{\Gamma_2^{\mathrm{O}}} P_{1i} n_i q \mathrm{d}\Gamma + I_{\Gamma_C} + I_{\mathrm{interface}} \tag{4}$$

where the closed paths $\Gamma_1^{\mathrm{O}} = \Gamma_\varepsilon^- + \Gamma_C^- + \Gamma_{B1} + \Gamma_I + \Gamma_C^+ + \Gamma_{B3}$ and $\Gamma_2^{\mathrm{O}} = \Gamma_{B2} + \Gamma_I^-$. I_{Γ_C} is a line integral along the crack faces and its expression is

$$I_{\Gamma_C} = \int_{\Gamma_C^- + \Gamma_C^+} P_{1i} n_i q \mathrm{d}\Gamma. \tag{5}$$

$I_{\mathrm{interface}}$ is a line integral with respect to the jump of P_{1i} along the interface Γ_I and its expression is

$$I_{\mathrm{interface}} = -\int_{\Gamma_I} \left[\left[P_{1i} \right] \right] n_i q \mathrm{d}\Gamma \tag{6}$$

where the double bracket symbol [[*]] denotes the jump of the quantity (*). The detailed derivations of Eq. (4) can refer to Yu et al. (2009).

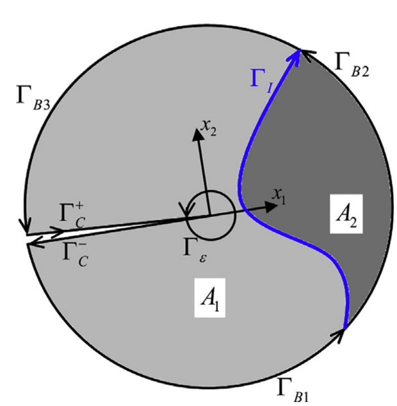

Fig. 2. Integral domain cut by an interface Γ_I (Region A_1: light gray; Region A_2: dark gray).