European Journal of Mechanics A/Solids 49 (2015) 531–538

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A new macroscopic criterion of porous materials with a Mises-Schleicher compressible matrix

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ARTICLE INFO

Article history:
Received 30 June 2014
Accepted 24 September 2014
Available online 12 October 2014

Keywords:
Porous materials
Mises-Schleicher
Macroscopic criterion

ABSTRACT

This study aims at determining the macroscopic strength of ductile porous materials with a Mises-Schleicher type compressible matrix. According to (Pastor et al., 2013b), there is no available criterion at the moment for this studied material. To this end, a new macroscopic criterion will be established in this paper by considering some special conditions. The newly derived criterion is then assessed and validated by comparing their predictions with numerical bounds (lower and upper bounds) proposed by (Pastor et al., 2013b). A good agreement is found not only for a Mises-Schleicher type matrix with weak asymmetry between tensile and compressive yield stresses, but also for a strongly asymmetric matrix with low and high porosities. The new macroscopic criterion improves the existing ones proposed by (Lee and Oung, 2000) and (Durban et al., 2010).

1. Introduction

For porous materials having a von Mises incompressible matrix, Gurson (Gurson, 1977) delivers a macroscopic yield criterion for spherical or cylindrical void by means of a limit analysis approach with a uniform strain rate boundary conditions. Then a huge amount of extensions for an incompressible matrix has been proposed based on this famous model: to better reproduce unite-cell simulations, (Tvergaard, 1981) and (Tvergaard and Needleman, 1984) (see also (Tvergaard, 1990)) proposed a heuristic extension of the Gurson model, known as the GTN model which is widely used in structural computations; works concerning with voids shape effects ((Gologanu et al., 1993), (Gologanu et al., 1994), (Gologanu et al., 1997), (Pardoen and Hutchinson, 2000), (Monchiet et al., 2008), (Monchiet et al., 2014)); taking into account the plastic anisotropy of matrix (Pan et al., 1997; Benzerga and Besson, 2001; Liao, 2004; Monchiet et al., 2008; Keralavarma and Benzerga, 2010) or tension-compression asymmetry (Cazacu and Stewart Joel, 2009). For some pressure-sensitive porous materials with plastic compressible matrix (such as geomaterials, rock, polymers, ceramics, etc.), the hydrostatic stress has a great influence on the yield surface of the matrix. To consider this effect, some studies have been devoted to porous materials with a Drucker-Prager type compressible matrix (see for instance (Jeong, 2002; Guo et al., 2008; Trillat and Pastor, 2005)); a Green type matrix (an elliptic isotropic criterion (Green, 1972)) has been recently studied by (Shen et al., 2012; Pastor et al., 2013a; Fritzen et al., 2013; Shen et al., 2013) to consider the compressibility in the matrix; some works focus on the double porous material with two populations of voids at different scales (Vincent et al., 2008, 2009; Shen et al., 2012; Shen et al., 2014). Concerning a pressure-sensitive matrix obeying the Mises-Schleicher criterion (Schleicher, 1926) (parabolic type), many works (such as (Raghava et al., 1973), (Zhang et al., 2008), (Kovrizhnykh, 2004), (Aubertin and Li, 2004)) have been done: (Lee and Oung, 2000) and (Durban et al., 2010) have proposed macroscopic criteria for porous materials with a Mises-Schleicher type matrix to consider the effects of porosity and the compressibility of the matrix on the macroscopic behavior; recently, (Monchiet and Kondo, 2012) established the closed-form expressions of the velocity and of the stress field for the case of hydrostatic tension and compression; (Pastor et al., 2013b) proposed numerical bounds (lower and upper bounds) for this class of pressure sensitive porous materials by means of 3D finite element formulations of the static and kinematic methods of Limit Analysis. Comparing with the numerical results, these criteria proposed by (Lee and Oung, 2000) and (Durban et al., 2010) can not be validated for high compressible matrix (strong asymmetry between tension and compression).

The purpose of this study is to propose a new macroscopic criterion for porous materials with a Mises-Schleicher matrix. The

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http://dx.doi.org/10.1016/j.euromechsol.2014.09.010
0997-7538/

paper is organized as follows: a closed-form expression of the macroscopic yield function is firstly established in Section 2. The new macroscopic criterion is then compared with existing ones proposed by (Lee and Oung, 2000) and (Durban et al., 2010) in Section 3. In Section 4, the obtained results are assessed and discussed by comparing them with recent numerical results (lower and upper bounds) of (Pastor et al., 2013b).

2. Macroscopic criterion for porous materials with a Mises-Schleicher matrix

2.1. Problem statement

A hollow sphere will be firstly considered in this section with the internal radii a and the external one b. $\mathbf{\Omega}$ denotes the volume of the unit cell, $|\Omega| = 4\pi b^3/3$, $\mathbf{\Omega}_m$ the matrix, whereas ω denotes the volume of the void, $|\omega| = 4\pi a^3/3$. The porosity of this hollow sphere is $f = a^3/b^3$. The matrix here is assumed isotropic and pressure sensitive, which obeys to the Mises-Schleicher criterion ((Schleicher, 1926), (Lubliner, 1990)):

$$f(\boldsymbol{\sigma}) = \sigma_{eq}^2 + 3\alpha\sigma_0\sigma_m - \sigma_0^2 \leq 0 \tag{1}$$

where $\sigma_m = \mathrm{tr}(\boldsymbol{\sigma})/3$ denotes the mean stress, $\sigma_{eq} = \sqrt{(3/2)\overline{\boldsymbol{\sigma}} : \overline{\boldsymbol{\sigma}}}$ is the equivalent stress with $\overline{\boldsymbol{\sigma}}$ the deviatoric part of the stress tensor, $\overline{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \sigma_m\mathbf{I}$ and $\mathbf{I}$ the second-order unit tensor. The plastic compressible parameter α and σ_0 are related to the tensile yield stress σ_T and to the absolute yield stress in compression σ_C ($\sigma_C \geq \sigma_T$) by:

$$\alpha = \frac{\sigma_C - \sigma_T}{\sqrt{\sigma_C\sigma_T}}, \quad \sigma_0 = \sqrt{\sigma_C\sigma_T} \tag{2}$$

In the case of hydrostatic loading, the local stress field in the solid phase of the rigid-plastic hollow sphere obeying to the Mises-Schleicher criterion has been derived in (Monchiet and Kondo, 2012):

$$\begin{aligned} \sigma_{rr} &= \frac{\sigma_0}{3\alpha}\left[1 - 2\,\alpha^2 W\left(\frac{pa^3}{r^3}\right) - \alpha^2 W^2\left(\frac{pa^3}{r^3}\right)\right] \\ \sigma_{\theta\theta} &= \frac{\sigma_0}{3\alpha}\left[1 + \alpha^2 W\left(\frac{pa^3}{r^3}\right) - \alpha^2 W^2\left(\frac{pa^3}{r^3}\right)\right] \end{aligned} \tag{3}$$

where W denotes the "Lambert W" function which satisfies $W(x)\,e^{W(x)} = x$. $W(x)$ has two branch: upper branch $W_0(x) \geq -1$ (for $x \geq -1/e$) and lower branch $W_{-1}(x) \leq -1$ (for $-1/e \leq x < 0$). In the case of tension, $W(px)$ in (3) is $W_0(p_+x)$, whereas in the case of compression, $W(px) = W_{-1}(p_-x)$. The coefficient p_+ and p_- are functions of α and the expressions are given:

$$\begin{cases} p_+ = z_+\exp(z_+), & z_+ = \dfrac{-\alpha + \sqrt{\alpha^2+1}}{\alpha} \\ p_- = z_-\exp(z_-), & z_- = \dfrac{-\alpha - \sqrt{\alpha^2+1}}{\alpha} \end{cases} \tag{4}$$

For a purely hydrostatic loading, the exact solution of the hollow sphere with a Mises-Schleicher compressible matrix is given by (Monchiet and Kondo, 2012):

$$\begin{cases} \dfrac{\Sigma_m}{\sigma_0} = \dfrac{1 - \alpha^2 W_0^2(fp_+) - 2\alpha^2 W_0(fp_+)}{3\alpha}, & \text{Tension} \\ \dfrac{\Sigma_m}{\sigma_0} = \dfrac{1 - \alpha^2 W_{-1}^2(fp_-) - 2\alpha^2 W_{-1}(fp_-)}{3\alpha}, & \text{Compression} \end{cases} \tag{5}$$

2.2. Formulation of the macroscopic criterion

In this section, a new closed-form expression of the macroscopic criterion will be established for the studied porous medium with a Mises-Schleicher compressible matrix. The searched macroscopic criterion should satisfy some conditions. For example, it should retrieve the Gurson's criterion when the plastic compressible parameter $\alpha \to 0$ (von Mises type matrix); the criterion of the solid matrix must be found when the porosity $f \to 0$, etc.. In order to establish the macroscopic criterion, some special conditions will be firstly considered:

- Condition 1: Purely hydrostatic loading ($\Sigma_{eq} = 0$);
- Condition 2: Compressible parameter $\alpha \to 0$;
- Condition 3: Porosity $f \to 0$;
- Condition 4: Deviatoric loading ($\Sigma_m = 0$).

The exact solutions have been obtained for hydrostatic tension and compression (Monchiet and Kondo, 2012). In order to guarantee the accuracy and to obtain a Gurson-like criterion, we will start from this special case to establish a macroscopic criterion for the studied porous materials.

- Case of purely hydrostatic loading (tension and compression):

To satisfy the condition 1, the searched macroscopic criterion should get the exact values of $\frac{\Sigma_m}{\sigma_0}$ under purely hydrostatic loadings ($\Sigma_{eq} = 0$):

$$\frac{\Sigma_m}{\sigma_0} = \frac{1 - \alpha^2 W^2(fp) - 2\alpha^2 W(fp)}{3\alpha} \tag{6}$$

where $W(fp) = W_{-1}(fp_-)$ for compression and $W(fp) = W_0(fp_+)$ for tension. The expressions of p_- and p_+ are given by (4).

The Equation (6) can also be rewritten as[1]:

$$\ln\left[\alpha^2 W^2(fp) + 2\alpha^2 W(fp)\right] = \ln\left(1 - 3\alpha\frac{\Sigma_m}{\sigma_0}\right) \tag{7}$$

When $\alpha \to 0$ for a von Mises type matrix, as indicated in (Monchiet and Kondo, 2012), the values of $\frac{\Sigma_m}{\sigma_0}$ provided by (6) or (7) are $-\frac{2}{3}\ln(f)$ for tension case and $\frac{2}{3}\ln(f)$ for compression case, which coincide with the results of Gurson. In the second step, the special case of incompressible matrix will be considered.

- Case of the compressible parameter $\alpha \to 0$ and $\Sigma_{eq} = 0$:

In this case, the criterion (1) reduces to the one of von Mises and there is non compressibility in the matrix. It is to say that the famous Gurson's criterion should be retrieved from the searched macroscopic criterion when $\alpha \to 0$. To this end, the following mathematical property is used for the appearance of the "cosh()" term as the one in Gurson's criterion:

$$\gamma = e^{-\vartheta} \Leftrightarrow 2\gamma\cosh\left(\vartheta\right) - 1 - \gamma^2 = 0 \tag{8}$$

For this purpose, the exact solution of purely hydrostatic loading (7) can be rewritten as the following form:

$$\left[\alpha^2 W^2(fp) + 2\alpha^2 W(fp)\right] = e^{\ln\left(1 - 3\alpha\frac{\Sigma_m}{\sigma_0}\right)} \tag{9}$$

[1] The value of $\alpha^2 W^2(fp) + 2\alpha^2 W(fp)$ is positive.