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Effects of non-uniform Winkler foundation and non-homogeneity on the free vibration of an orthotropic elliptical cylindrical shell

Mousa Khalifa

Department of Mathematics, Faculty of Science at Qena, South Valley University, Egypt

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ABSTRACT

Flügge's shell theory and solution for the vibration analysis of a non-homogeneous orthotropic elliptical cylindrical shell resting on a non-uniform Winkler foundation are presented. The theoretical analysis of the governing equations of the shell is formulated to overcome the mathematical difficulties of mode coupling of variable curvature and homogeneity of shell. Using the transfer matrix of the shell, the vibration equations based on the variable Winkler foundation are written in a matrix differential equation of first order in the circumferential coordinate and solved numerically. The proposed model is applied to get the vibration frequencies and the corresponding mode shapes of the symmetrical and antisymmetrical vibration modes. The sensitivity of the vibration behavior and bending deformations to the non-uniform Winkler foundation moduli, homogeneity variation, elliptical and orthotropy of the shell is studied for different type-modes of vibrations.

1. Introduction

Materials and structural components are often non-homogeneous, either by design or because of the physical composition and imperfections in the underlying materials. Elliptical shells represent one of the principal elements of aerospace and marine structures, pressure vessels, process equipment and piping. The study of mechanical behavior of vibration for these shells can help designer achieve a reduction in the weight and an increase in the stiffness, especially when those are in contact with elastic foundations. Generally, the frequencies and mode shapes of vibration depend on some determining functions such as the radius of the curvature of the neutral surface, the shell thickness, the shape of the shell edges, the elastic media, and the elastic properties of shell and so on. In simple cases when these functions are constant without foundations, the vibration deflection displacements occupy the entire shell surface. If the determining functions vary from point to point of the neutral surface then localization of the vibration modes lies near the weakest lines on the shell surface. Mathematically, the consideration of non-homogeneity, orthotropy, variable elastic foundation and aspect ratio leads to a very complex problem involving several parameters. So, numerical or approximate techniques are necessary for their analysis. Since more attention is being paid to the analysis of shell's behavior embedded in elastic foundations through the Winkler–Pasternak model. There are different approaches to analyze the interaction between a structure and an ambient medium, see Pasternak (1954), Kerr (1964) and Bajenov (1975). However, very few models representing the behavior of non-homogeneous materials have been reported in the literature. Vibrations of shells on elastic foundations have been studied only recently and focused on the circular cylindrical shell case; see Paliwal and Bhalla (1993), Paliwal et al. (1996), Paliwal and Pandey (1998, 2001), Paliwal and Singh (1999), Ng and Lam (2000) and Gunawan et al. (2004, 2006). In most of these studies, the authors investigate the vibrations of homogeneous isotropic and orthotropic cylindrical shells on an elastic foundation using membrane theory under effect of Winkler and Pasternak type foundations. On the other hand, the study of vibration of circular and non-circular cylindrical shells of non-homogenous materials is very scarce, but there are a few important publications related to this study such as Sofiyev and Keskin (2004) and Sofiyev et al. (2010, 2011). As it is found recently, a few researchers are directed to devote their studies for the vibration behavior of functionally graded elastic cylindrical and conical shells in addition to panel shells resting on elastic foundations such as Shah et al. (2010), Sofiyev and Kuruoglu (2012), Najafov et al. (2012), Tornabene (2011), Tornabene and Ceruti (2013) and they are employed wave propagation method and Galerkin method to solve the dynamical equations. In contrary, despite the great value of engineering applications for the elliptical shells there are no previous considerations for studying the vibration behavior for such

E-mail address: mossa@dr.com.

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shells under the effect of elastic foundations due to the difficulties of mode coupling of terms in the solution model. But there are some important studies of vibration for such shells without any media affected, see those Sewall and Pusey (1971), Yamada et al. (1985) and Soedel (2004) who are used the thin-shell theory in their analysis. In spite of extensive works, which have been carried out, it is felt that the combined effects of the orthotropy, homogeneity variation, and the existence of elastic foundation on the vibration characteristics of elliptical cylindrical shells have not been analyzed. In the current study, an attempt is made to address this problem. The objective of this paper is to find the natural frequencies and corresponding mode shapes of vibration for non-homogenous orthotropic elliptical cylindrical shells resting on a variable Winkler foundation based on the Flügge's shell theory and using the transfer matrix approach with Romberg integration method. The results reveal that the variations of the elliptical cylindrical shell parameters have significant effects on the values of the natural frequencies and the mode shapes.

2. Theory and mathematical model of the problem

It is known for researchers that the study of vibration problems in shells depends on the shell geometry and the model of shell theory. The present problem is modeled by Flügge's theory in existence of a Winkler-type foundation. This part is presented as follows:

2.1. Geometric formulation

The present shell is an orthotropic elliptical cylindrical shell and its material is non-homogeneously elastic. The curvature of cross-section profile of the elliptical shell is defined by the equation $r = r_0 f(\theta)$, where r is the local varied radius along the cross-section mid-line, r_0 is the reference radius of curvature, chosen to be the radius of a circle having the same circumference as the elliptical profile, and $f(\theta)$ is a prescribed function of θ and can be taken as:

$$f(\theta) = \frac{A}{\sqrt{1 + \varepsilon \cos^2 \theta}}, \quad 0 \le \theta \le 2\pi, \tag{1}$$

where ε is the elliptical parameter and measures the eccentricity of the cross section of the shell based on (A/B) value, and expressed by the semi-major, a, and semi-minor, b, axes of the elliptical cross-section profile as:

$$\varepsilon = \frac{a^2}{b^2} - 1, \ B = b/r_0 \text{ and } A = a/r_0. \tag{2}$$

The position of a point on the middle surface of the shell is defined by the cylindrical coordinates (x, s, z), as shown in Fig. 1. The displacements of the middle surface of the shell are denoted by u, v and w in the axial, circumferential and transverse directions, respectively. The shell geometry is described by H, the thickness and L, the axial length of the shell. If we suppose that the axial and circumferential directions are principal axes of the orthotropic material. Up to this point, the next relation is obtained:

$$\nu_x E_s = \nu_s E_x, \tag{3}$$

where E_x, E_s are Young's moduli and ν_x, ν_s are Poisson's ratios in the axial and circumferential directions, respectively.

2.2. Modal radius of curvature

Since the solution of the present problem drastically depends on the variable curvature $r(\theta)$, it follows that the radius of curvature should represent the actual geometry of the middle surface of the shell. In the present study, the radius of curvature can be got from this equation:

$$R(\theta)/r_0 = \left(r^2 + r^{\bullet 2}\right)^{3/2} / r_0\left(r^2 + 2r^{\bullet 2} - rr^{\bullet\bullet}\right). \tag{4}$$

Using Eqs. (1) and (4), the expression $C = R(\theta)/r_0$ which will be appeared in Eqs. (25), can be found to take the form:

$$C = A(a/b)\left(\frac{1 - \varepsilon_1 \cos^2 \theta}{1 - \varepsilon_2 \cos^2 \theta}\right)^{3/2}, \quad \varepsilon_1 = 1 - (b/a)^2, \quad \varepsilon_2 = 1 - (b/a)^4 \tag{5}$$

which represents the dimensionless radius of curvature of the elliptical cylindrical shell.

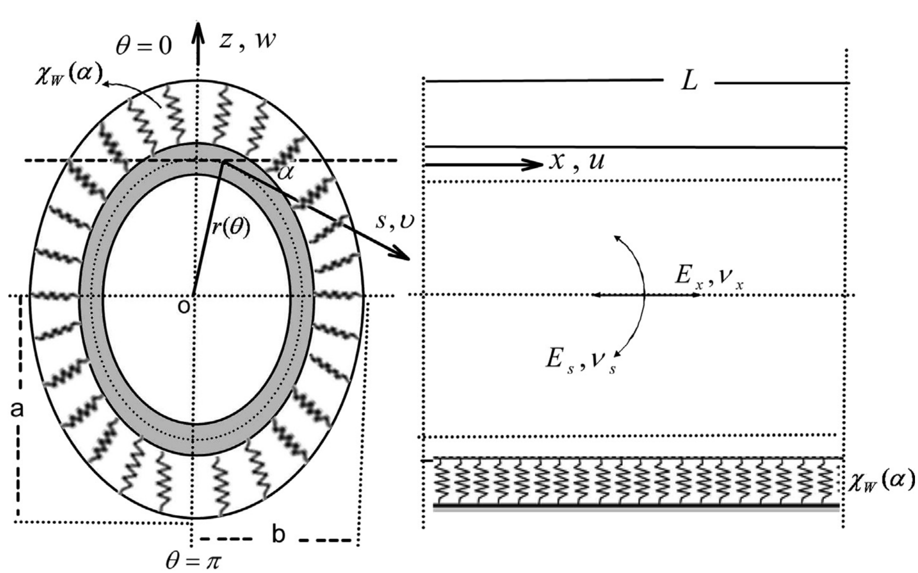

Fig. 1. Coordinate system and geometry of an elliptical orthotropic cylindrical shell resting on a non-uniform Winkler foundation.