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Experimental characterization, modeling and parametric identification of the non linear dynamic behavior of viscoelastic components

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ABSTRACT

The aim of this paper is to investigate non linear dynamic behavior of viscoelastic components. The dynamic characteristics of viscoelastic components depend on frequency, amplitude, preload, and temperature. A Non Linear Generalized Maxwell Model (NLGMM), with only 4 independent parameters, is proposed. The NLGMM is based on the separation between the linear viscoelasticity and the non linear stiffness. This assumption is validated on a large range of experimental measures. The NLGMM can be used for different kinds of excitations in non linear dynamics: periodic, transient or random excitations. The non linear stiffness is represented by a simple polynomial function. Comparisons between measures and computed values have been carried on several viscoelastic samples. The NLGMM shows a good accordance with experimental results.

1. Introduction

Viscoelastic components, which have substantial energy absorption abilities, are always incorporated into automobile and aerospace structure systems in order to damp mechanical vibrations and thus avoid serious damage. Viscoelasticity is widely studied since decades: considering works of Ferry (1961), Vinh (1967), Caputo and Mainardi (1971), Lakes (1999), Chevalier and Vinh (2010) and Balmès and Leclère (2009). Viscoelasticity is a causal phenomenon for which the force always precedes the displacement. This behavior can be described by the relaxation function or the creep function. In the Fourier domain, the dynamic stiffness is a complex function which depends on frequency.

Several experimental studies have been carried out to characterize viscoelastic behavior providing important results and understanding of viscoelastic components dynamics. Oberst and Frankenfeld (1952) proposed to study the first mode of a sandwich beam consisting of metal skins and a viscoelastic core. Their method allows knowing the damping induced by the viscoelastic core at the frequency of the mode. Several authors like Barbosa and Farage (2008) and Castello et al. (2008) used this kind of technique for viscoelastic parameters identification. It is also possible to deduce the mechanical properties of a viscoelastic component from different measures of natural frequencies of a simple form sample like a beam for example, see Chevalier (2002). These methods use Frequency Response Function (FRF), hence, they can only characterize the frequencies of modes and not on a wide frequency band. Moreover, these methods are valid under the assumption of linear material excitation amplitude. Chen (2000) suggested measuring directly the relaxation functions and creep to deduce the coefficients of a series of Prony. However, this is very efficient to get values at low frequency, when the material takes time to respond to the excitation. But to get high frequency values, a perfect unit step function is required to assess when exciting the material, which is technically hard. The most suitable kind of test is the Dynamic Mechanical Analysis (DMA), it is a useful technique for acquiring knowledge on the behavior of a material versus frequency. A DMA tester is used in this work to determine the dynamic stiffness of the viscoelastic component depending on the frequency.

Moreover, viscoelastic components are a key element in designing desired dynamic behavior of mechanical systems; therefore, different models describing viscoelastic behavior have been developed. Gaul et al. (1991) presented the constant complex

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modulus model which is non-causal model, it is only suitable in the frequency domain, but, it is not a relevant model since its modulus is constant. Maxwell model represented by Park (2001) as a spring and dash-pot connected in series and Kevin Voigt model which consists of a spring and dash-pot in parallel, are efficient only on a small frequency range. In fact, they are unrealistic respectively at low and high frequencies, where their modulus is respectively: infinitely small and high and the dynamic stiffness phase angle of the Kevin Voigt model is linearly dependent of frequency. The Zener model, see Huynh et al. (2002), underestimates the dynamic stiffness at low frequencies and overestimates it at high frequencies. Just as the Kevin Voigt model, the Zener model is unable to capture the frequency dependence of the phase angle. Koeller (1984) used Generalized Maxwell Model, which would refer to a spring in parallel with respectively Maxwell cells, to describe the frequency dependence of dynamic stiffness of the viscoelastic components. However the dynamic characteristics of viscoelastic components are often very complex in nature, due to the fact that the response is dependent not only on frequency but also several variables, such as amplitude, preload, and temperature which can be certainly critical in capturing the mechanical proprieties and non linear dynamical behavior appears. Consequently, various methods treating viscoelastic non linear dynamics have been developed. Volterra model, see Schetzen (1980), is used in the work of Saad (2003) to predict amplitude dependency observed experimentally and to linearize a visco-hyperelastic model to take preload effects into account. The non linear dynamic behavior of preloaded multilayer plates incorporating visco-hyperelastic material confined between stiff layers and worked as a damping layer is investigated by Gacem (2007). Monsia (2011) proposed a non linear generalized Maxwell model which consisting of a non linear spring connected in series with a non linear dash-pot obeying a power law with constant material parameters, for representing the time-dependent properties of a variety of viscoelastic materials. Monsia (2012) developed a non linear mathematical model with constant material coefficients applicable for characterizing the time-dependent deformation behavior of a variety of materials under a constant loading.

In this context, this paper introduced a new approach for non linear Generalized Maxwell Model in order to describe the dynamic behavior of viscoelastic components. DMA tests have been conducted in order to identify parameters of the proposed NLGMM which shows a good accuracy when a comparison between experiments and simulations is performed.

The planning of the present paper is as follows: in Section 2, a description of the experimental procedure to characterize the viscoelastic component is presented. The proposed NLGMM and the identification techniques of its linear and non linear parameters are detailed in Section 3. Comparison between identified and measured values is also performed. In Section 4, the validity of the NLGMM is investigated and discussed.

2. Experimental characterization

When a material is subjected to a sinusoidal cyclic displacement of angular frequency ω:

$$x(t) = x_{00}\sin(\omega t) \tag{1}$$

The term x_{00} represents the displacement amplitude.

The force response is sinusoidal at the same frequency but with a dephasing angle φ, called loss angle:

$$F(t) = F_{00}\sin(\omega t + \varphi) \tag{2}$$

The term F_{00} represents the force amplitude.

Generally, this assumption, called the first harmonic, is not sufficient. Typically, the force response contains higher order harmonics, and the real response is expressed as follows, see Long (2005):

$$F(t) = \sum_k F_k \sin(k\omega t + \varphi_k) \tag{3}$$

In the case of the assumption of the first harmonic, the complex stiffness $K^*(\omega)$ relates the Fourier transform of the imposed displacement $\hat{x}(\omega)$ to the corresponding force $\hat{F}(\omega)$ is defined as follows:

$$\hat{F}(\omega) = K^*(\omega)\hat{x}(\omega) \tag{4}$$

with the Fourier transform:

$$\hat{x}(\omega) = \int_{-\infty}^{+\infty} x(t)\exp(-j\omega t)\mathrm{d}t \tag{5}$$

$$\hat{F}(\omega) = \int_{-\infty}^{+\infty} F(t)\exp(-j\omega t)\mathrm{d}t \tag{6}$$

The dynamic stiffness is defined as:

$$K^*(\omega) = \frac{\hat{F}(\omega)}{\hat{x}(\omega)} = \frac{F_{00}}{x_{00}}\cdot\exp(j\varphi) = K'(\omega) + jK''(\omega) = K'(\omega)[1 + j\tan\varphi] \tag{7}$$

Where K' is the real part of dynamic stiffness andK''its imaginary part.

2.1. Experiment

The DMA tester is a bench test performed to characterize the behavior of dynamic compression of a rubber sample as shown in Fig. 1.

The cylindrical elastomeric sample is subjected to uniaxial compression tests. The mechanical solicitation is performed using a hydraulic cylinder with a LVDT (Linear Variable Differential Transformer) displacement sensor. The system is also equipped with a force sensor built into the base of the assembly apparatus. The force and displacement signals after analog conditioning are returned on a spectrum analyzer for digital processing. The entire system is controlled by a computer equipped with an interface GPIB (General Purpose Interface Bus) card connected to the FFT (Fast Fourier Transform) analyzer which manages and controls the sweeping frequencies of the signal by incrementing the excitation frequency for each acquisition.

To determine dynamic stiffness K^* of a material, the sample is placed between two rigid surfaces. Surfaces are flat and parallel as presented in Fig. 2.

During these unidirectional tests, devices measure the vertical force imposed and ΔH the vertical displacement of upper surface of the rubber sample. For a cylindrical rubber sample of height $H_0 = 13$ mm and diameter $D_0 = 28$ mm, the surface on which the force acts is $S_0 = \pi \times (D_0/2)^2$ The dynamic stiffness is given by:

$$K^* = \frac{F}{\Delta H} \tag{8}$$

Tests are carried out to evaluate the dynamic behavior of elastomeric sample and are performed by applying a mechanical sinusoidal solicitation. Generally, elastomeric material presents a