European Journal of Mechanics A/Solids 42 (2013) 395–401

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Causality in the Bauschinger effect generation and in other deformation processes in metals

V. Kafka [a], D. Vokoun [b,*]

[a] Institute of Theoretical and Applied Mechanics ASCR, v.v.i., Prosecka 76, 19000 Prague, Czech Republic
[b] Institute of Physics ASCR, v.v.i., Na Slovance 2, 18221 Prague, Czech Republic

ARTICLE INFO

Article history:
Received 20 February 2013
Accepted 16 July 2013
Available online 2 August 2013

Keywords:
Bauschinger effect
Mesomechanics
Internal variables

ABSTRACT

The phenomenon of Bauschinger effect and of other generally known deformation processess of metals is explained and modeled on the mesoscale. The basic feature of the model consists in the use of tensorial internal variables that have explicit physical meaning – internal mesomechanical stresses. The metallic materials under consideration are modeled as two-phase media with two substructures, one compliant and the other resistant. It is shown that Bauschinger effect and other deformation phenomena observed in most metallic materials can be explained and described as results of interplays of these substructures.

1. Introduction

The attention paid to yield surfaces is very old, although in last decades more attention is paid to fracture mechanics. In spite of it, the importance of yield criteria remains. Achieving the elastic limit given by a yield surface does not mean achieving the strength limit of the respective element, but it can mean achieving the strength limit of the construction, in which this element is embedded. Therefore, it still holds that remaining in the elastic state the safest policy for considering safety of constructions and the study of the following plastic deformation is still an important topic.

The aim of the current study is not a mere description of the experimentally observed phenomena related to yield surface changes, but mainly discussion why these phenomena arise as they do, and modeling the respective processes.

The most common and commonly used yield criterion is Mises' criterion (Von Mises, 1913). Its classical – somewhat cumbersome – form reads:

$$\frac{1}{\sqrt{2}}\Big[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2 \\ +6\left(\tau_{12}^2+\tau_{23}^2+\tau_{31}^2\right)\Big]^{\frac{1}{2}}\le\sigma_y \quad (1)$$

and is interpreted as a cylinder in a principal stresses coordinate system that makes the same angle with the three principal axes.

With the use of Einstein's notation, Hill (1950) suggested a more elegant form of this criterion:

$$s_{ij}s_{ij}\le 2k^2\left(=\frac{2}{3}\sigma_y^2=\frac{3}{2}s^2\right) \quad (2)$$

where s_{ij} is the deviatoric part of the stress tensor σ_{ij}, ($s_{ij}=\sigma_{ij}-\delta_{ij}\sigma$, $\sigma=1/3\sigma_{ii}$), $\delta_{ij}\sigma$ is its isotropic part, δ_{ij} the Kronecker delta, k is a limit value in the case of sheer stress loading, $\sigma_y[s]$ is a limit value of principal stress σ_{11} [of its deviatoric part s_{11}] in the case of uniaxial tensile loading. It is evident that this criterion is independent of the isotropic part $\delta_{ij}\sigma$ of the stress tensor, called sometimes hydrostatic pressure. Equation (2) can be interpreted as a hyper-sphere, as it resembles the equation of a sphere written with the use of Einstein's notation ($x_ix_i\le r^2$).

The Mises criterion (similarly as Tresca's shear stress criterion) is meant for isotropic materials. For anisotropic materials, Hill (1948, 1950) suggested – by mere analogy – the following form:

$$F(\sigma_{22}-\sigma_{33})^2+G(\sigma_{33}-\sigma_{11})^2+H(\sigma_{11}-\sigma_{22})^2+2L\sigma_{23}^2 \\ +2M\sigma_{31}^2+2N\sigma_{12}^2=1 \quad (3)$$

where F, G, H, L, M, N are constants to be determined experimentally. It is straightforward to transform this equation to a form that comprises only deviatoric components:

* Corresponding author. Tel.: +420 266052427.
E-mail address: vokoun@fzu.cz (D. Vokoun).

0997-7538/$ – see front matter © 2013 Elsevier Masson SAS. All rights reserved.
http://dx.doi.org/10.1016/j.euromechsol.2013.07.008

Nomenclature

General mechanics

σ_{ij}	stress tensor
$\delta_{ij}\sigma$	isotropic part of $\sigma_{ij}(\sigma = \sigma_{ii}/3)$
s_{ij}	deviatoric part of $\sigma_{ij}(\sigma_{ij}-\delta_{ij}\sigma)$
ε_{ij}	strain tensor
$\delta_{ij}\varepsilon$	isotropic part of $\varepsilon_{ij}(\varepsilon = \varepsilon_{ii}/3)$
e_{ij}	deviatoric part of $\varepsilon_{ij}(\varepsilon_{ij}-\delta_{ij}\varepsilon)$
δ_{ij}	Kronecker's delta
E	Young's modulus
ν	Poisson's ratio
$\mu=(1+\nu)/E$	deviatoric elastic compliance
$\rho=(1-2\nu)/E$	isotropic elastic compliance

Specific symbols

$\bar{I}$	overbar that relates symbol I to its macroscopic value – average in the representative volume element (RVE)
I^{ω}	superscript ω relates symbol I to the ω- constituent – average in the subvolume of RVE that is filled in by the ω- constituent
$\omega = c$	compliant constituent in the general two-phase model
$\omega = r$	resistant constituent in the general two-phase model
$\vert_{ij}$	Einstein's notation
	$\varepsilon'_{ij} = \varepsilon_{ij} - \bar{\varepsilon}_{ij}$
$\delta_{ij}\varepsilon'$	isotropic part of ε'_{ij}
e'_{ij}	deviatoric part of ε'_{ij}
σ'_{ij}	stress related to ε'_{ij} similarly as is σ_{ij} related to ε_{ij}
$\delta_{ij}\sigma'$	isotropic part of σ'_{ij}
s'_{ij}	deviatoric part of σ'_{ij}
$v^c[v^r]$	volume fraction of the compliant [resistant] constituent in the two-phase model
$k[\sigma_y]\{s\}$	yield limit for: sheer stress loading [σ_{11} in the case of uniaxial tensile loading in the x_1-direction] {s_{11} in the case of uniaxial tensile loading in the x_1-direction}
η^{ω}	structural parameter ($\omega = c, r$)
p	$v^c\eta^c + v^r\eta^r$
q	$p + \eta^c\eta^r$
$\dot{h}^c$	0 in the case of elasticity, [$= \mathrm{d}\lambda^c/\mathrm{d}t$ in the case of plasticity], { $= 1/2H^c$ in the case of rheological deformation}
$\mathrm{d}\lambda^c$	increment of scalar measure of plastic deformation in the compliant constituent
H^c	coefficient of viscosity of the compliant constituent

$$F(s_{22}-s_{33})^2+G(s_{33}-s_{11})^2+H(s_{11}-s_{22})^2+2Ls_{23}^2+2Ms_{31}^2 +2Ns_{12}^2=1 \tag{4}$$

Hence, this criterion is again independent of isotropic stress.

However, the criteria that are independent of isotropic stress are not appropriate for all materials and a number of different phenomenological approaches to the description of more complicated forms of yield criteria have been suggested in the past (Caddell et al., 1974; Kafka, 1987; Bigoni and Piccolroaz, 2004).

A topic closely related to yield surfaces is Bauschinger effect. The interest in Bauschinger effect and yield criteria is quite old, but still alive (Lin et al., 1972; Caddell et al., 1973; Hill, 1979, 1993; Zhu et al., 1987; Chu, 1995; Deshpande et al., 2001; Alexandrov and Hwang, 2011; Francois, 2001; Liu et al., 2011; Vicente Alvarez, Bergant and Perez, 2010; Yilamu et al., 2010; Wang and Jia, 2011; Zhu et al., 2011; Bastun, 2012).

In the current study, we limit ourselves to Mises' criterion (2) and to the causality of its changes due to inelastic deformation: Bauschinger effect, plastic deformation, creep, relaxation, yield-point drop, serrated stress–strain diagram, Lüders' bands and necking. Our aim is first of all explanation and modeling the inner process that leads to Bauschinger effect, a phenomenon known for more than hundred years and observed in most metallic materials. It seems meaningless to look for experimental results received for one material and show that Bauschinger effect is valid for this one material, as Bauschinger effect is valid generally. Our aim is explanation and modeling of these generally known phenomena, not comparison with experimental results found for special materials. It would be another concept to choose one specific material and describe in detail all its properties.

Our approach uses tensorial internal variables *with clear physical meaning*, which are mesoscopic internal stresses. The first attempt to explain Bauschinger effect by internal stresses was published by Vasilev (1959), but was too superficial to receive much attention.

2. General model of the concept

Our analysis is based on the general concept of the first author (Kafka, 2001), in which the material under discussion is modeled as a two-phase medium, in which one phase is compliant (superscript c), the other resistant (superscript r). In applications to polycrystalline metals, the compliant phase corresponds to inner parts of grains with easy glide, the resistant phase to the rest of the material (impurities, precipitates, dislocations, boundary regions of grains with irregular atomic structure, etc.). In different materials and processes, the term "compliant" can mean plastic time-independent or rheological time-dependent deformation. The term "resistant" can mean restriction to elastic deformation, to elastic–plastic deformation with higher yield limit or of elastic-fracturing deformation. Apart from these qualitative characteristics, every material constituent is characterized by its Young's modulus, Poisson's ratio, volume fraction, structural parameters, yield criterion or strength criterion. Debonding of the material constituents is not taken into consideration; we do not have in mind composite materials, where this process can be important.

In our works, this *mesoscale* model has successfully been applied first of all to metallic materials, but also to concrete, polymers and biological tissues (Kafka, 1979a, 1979b, 1984, 1987, 2001, 2001a, 2008). Its basic set of equations reads:

$$v^r\sigma^r_{ij} + v^c\sigma^c_{ij} = \bar{\sigma}_{ij} \tag{5}$$

$$v^r\varepsilon^r_{ij} + v^c\varepsilon^c_{ij} = \bar{\varepsilon}_{ij} \tag{6}$$

$$\dot{e}^r_{ij} = \mu^r\dot{s}^r_{ij}, \quad \varepsilon^r = \rho^r\sigma^r \tag{7}$$

$$\dot{e}'^r_{ij} = \dot{e}^r_{ij} - \dot{\bar{e}}_{ij}, \quad \varepsilon'^r = \varepsilon^r - \bar{\varepsilon} \tag{8}$$

$$\dot{e}'^r_{ij} = \mu^r\,\dot{s}'^r_{ij}, \quad \varepsilon'^r = \rho^r\varepsilon'^r \tag{9}$$

$$\dot{e}^c_{ij} = \mu^c\dot{s}^c_{ij} + s^c_{ij}\dot{h}^c, \quad \varepsilon^c = \rho^c\sigma^c \tag{10}$$

$$\dot{e}'^c_{ij} = \dot{e}^c_{ij} - \dot{\bar{e}}_{ij}, \quad \varepsilon'^c = \varepsilon^c - \bar{\varepsilon} \tag{11}$$

$$\dot{e}'^c_{ij} = \mu^c\dot{s}'^c_{ij} + s'^c_{ij}\dot{h}^c, \quad \varepsilon'^c = \rho^c\varepsilon'^c \tag{12}$$