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An electrically impermeable and magnetically permeable interface crack with a contact zone in a magnetoelectroelastic bimaterial under uniform magnetoelectromechanical loads

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An interface crack with a frictionless contact zone at the right crack tip between two dissimilar magnetoelectroelastic materials under the action of remote mechanical, electrical and magnetic loads is considered. The open part of the crack is assumed to be electrically impermeable and magnetically permeable. Both the Dirichlet–Riemann boundary value problem and Hilbert problem have been formulated and solved exactly. Stress, electrical displacement and magnetic induction intensity factors as well as energy release rate are found in analytical forms. Transcendental equations and a closed form analytical formula for the determination of the real contact zone length have been derived and analyzed. Some numerical results are plotted to show the effects of the applied loads on the contact zone length, stress intensity factor and energy release rate.

1. Introduction

Magnetoelectroelastic materials have been widely used in electronics industry. The technical applications may include waveguides, sensors, phase invertors, transducers, etc. (Parton and Kudryavtsev, 1988). In the design of magnetoelectroelastic structures, it is important to take into account imperfections, such as cracks, which are often pre-existing or are generated by external loads during the service life. Therefore, in recent years, the research on fracture mechanics of magnetoelectroelastic materials has drawn a lot of interest (Zhou et al., 2004, 2009; Gao et al., 2004; Chue and Liu, 2005; Hu and Li, 2005; Feng et al., 2005; Feng and Su, 2006; Li and Kardomateas, 2006; Li and Lee, 2008; Niraula and Wang, 2006; Wang et al., 2006, 2008; Zhao et al., 2006; Feng et al., 2007; Yong and Zhou, 2007; Zhong and Zhang, 2010; Li, 2001; Singh et al., 2009).

For two-dimensional (2-D) plane crack problems, Liu et al. (2001) derived the Green's functions for an infinite magnetoelectroelastic plane containing an elliptic cavity. They reduced the cavity solution to obtain the solution for a permeable crack. Gao et al. (2003a,b) analyzed single and collinear cracks in an infinite magnetoelectroelastic material and obtained the extended stress intensity factors. Song and Sih (2003) and Sih et al. (2003) investigated the influence of both magnetic field and electrical field on crack growth, in particular, on crack initiation angle under various crack surface conditions for Mode-I, Mode-II, and mixed-mode crack models. Tian and Gabbert (2004) and Tian and Gabbert (2005) studied the interaction problem of multiple arbitrarily oriented and distributed cracks and the interaction problem of macrocrack-microcrack in homogeneous magnetoelectroelastic materials, respectively. Wang and Mai (2007) discussed the effects of four kinds of ideally magnetoelectrical crack face conditions on fracture properties of magnetoelectroelastic materials. Zhong and Li (2007) obtained the T-stress for a Griffith crack in an infinite magnetoelectroelastic medium based on magnetic and electrical boundary conditions nonlinearly dependent on the crack opening displacement. Zhou et al. (2007, 2008) investigated the static fracture behaviors of a single crack or two cracks in piezoelectric/piezomagnetic materials by the Schmidt method. Chen (2009) considered the energy release rate and path-independent integral in dynamic fracture of magneto-electro-thermo-elastic solids. Zhong et al. (2009) investigated the transient response of a magnetoelectroelastic solid with two collinear dielectric cracks under impacts.

However, all the above-mentioned works are related to crack in a homogenous magnetoelectroelastic medium. Due to the oscillating singularity of crack tips (Williams, 1959; Rice, 1988), the study of interface crack between dissimilar magnetoelectroelastic

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materials is very limited. Gao et al. (2003c) and Gao and Noda (2004) derived the exact solution for a permeable interface crack between two dissimilar magnetoelectroelastic solids under general applied loads and under uniform heat flow, respectively. Li and Kardomateas (2007) investigated the interface crack problem of dissimilar piezoelectromagneto-elastic anisotropic bimaterials under in-plane deformation taking the electric-magnetic field inside the interface crack into account. Feng et al. (2009, 2010) considered both the dynamic and static fracture problems of interface cracks between two dissimilar magnetoelectroelastic layers. Li et al. (2009) analyzed the magnetoelectroelastic field induced by a crack terminating at the interface of a bi-magnetoelectrical material. It is worth to mention that recently Zhao et al. (2008) further gave an analysis method of planar interface cracks of arbitrary shape in three-dimensional (3-D) transversely isotropic magnetoelectroelastic bimaterials, and that Zhu et al. (2010) investigated the mixed-mode stress intensity factors of 3-D interface crack in fully coupled magneto-electro-thermo-elastic multiphase composites, where the extended hyper-singular intergro-differential equation (E-HIDE) method was used.

On the other hand, as well known, by introducing contact zone model, the oscillating singularity can be effectively eliminated (Comninou, 1977; Atkinson, 1982; Simonov, 1985; Dundurs and Gautesen, 1988). About ten years ago, Qin and Mai (1999), Herrmann and Loboda (2000) and Herrmann et al. (2001) developed the contact zone model to interface crack problems of piezoelectric bimaterials. However, to the best of our knowledge, up till now, although lots of achievements on crack problems of magnetoelectroelastic materials have been made, because of mathematically complexity, only one paper related to contact zone model for an interface crack between two dissimilar magneto-electroelastic materials (Herrmann et al., 2010) was reported, where two kinds of magnetoelectrical boundary conditions, i.e., magnetoelectrically permeable, electrically permeable and magnetically impermeable, were considered.

In the present paper, we further analyze the interface crack problem by considering the contact zone model. Different from the work of Herrmann et al. (2010), the electrically impermeable and magnetically permeable crack surface assumption is adopted here. After a complex mathematics manipulation, all the contact zone length, field intensity factors (including stress, electrical displacement and magnetic induction intensity factors) and energy release rate are derived, and numerical results are given and analyzed in detail.

2. Basic equations for a magnetoelectroelastic solid

In the rectangular Cartesian coordinate system x_i $(i = 1, 2, 3)$, the governing equations for magnetoelectroelastic materials may be written in the following form (Gao and Noda, 2004):

$$\begin{cases} \sigma_{ij} = c_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s, \\ D_i = e_{iks}\varepsilon_{ks} + \alpha_{is}E_s + d_{is}H_s, \\ B_i = h_{iks}\varepsilon_{ks} + d_{is}E_s + \mu_{is}H_s, \end{cases} \tag{1}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \;\; E_i = -\varphi_{,i}, \;\; H_i = -\phi_{,i}, \tag{2}$$

$$\sigma_{ij,j} = 0, \;\; D_{i,i} = 0, \;\; B_{i,i} = 0, \tag{3}$$

where σ_{ij}, D_i, B_i are the components of the stresses, electrical displacements and magnetic inductions, respectively; ε_{ij}, E_i, H_i are the components of strains, electrical fields and magnetic fields, respectively; u_i, φ, ϕ are the mechanical displacement components, electrical potential and magnetic potential, respectively. c_{ijks}, e_{iks}, h_{iks}, d_{is} are the elastic, piezoelectric, piezomagnetic, and electromagnetic constants, respectively; α_{is}, μ_{si} are the dielectric permittivities and magnetic permeabilities, respectively. i, j, k, s range in {1, 2, 3}, the repeated indexes imply summation, and the comma stands for the differentiation with respect to the corresponding coordinate variables.

From Eqs. (1)–(3), one gets the following governing equations:

$$\begin{cases} \left(c_{ijks}u_k + e_{sij}\varphi + h_{sij}\phi\right)_{,si} = 0, \\ (e_{iks}u_k - \alpha_{is}\varphi - d_{is}\phi)_{,si} = 0, \\ (h_{iks}u_k - d_{is}\varphi - \mu_{is}\phi)_{,si} = 0. \end{cases} \tag{4}$$

By using the Lekhnitskii–Eshelby–Stroh representation and its application to magnetoelectroelastic materials, a general solution of Eq. (4) can be presented in the form (Gao and Noda, 2004)

$$\mathbf{V} = \mathbf{Af}(z) + \overline{\mathbf{A}}\overline{\mathbf{f}}(\bar{z}), \tag{5}$$

$$\mathbf{t} = \mathbf{Bf}'(z) + \overline{\mathbf{B}}\overline{\mathbf{f}}'(\bar{z}), \tag{6}$$

where $\mathbf{V} = [u_1, u_2, u_3, \varphi, \phi]^{\mathrm{T}}$, $\mathbf{t} = [\sigma_{31}, \sigma_{32}, \sigma_{33}, D_3, B_3]^{\mathrm{T}}$ (the superscript 'T' stands for the transposed matrix), $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4, \mathbf{A}_5]^{\mathrm{T}}$; $\mathbf{f}(z) = [\mathbf{f}_1(z_1), \mathbf{f}_2(z_2), \mathbf{f}_3(z_3), \mathbf{f}_4(z_4), \mathbf{f}_5(z_5)]^{\mathrm{T}}$ in Eqs. (5) and (6) is an arbitrary analytic vector function with five components determined later. $z_j = x_1 + p_j x_3$ $(j = 1, 2, \ldots, 5)$. For a fixed j, p_j and $\mathbf{A}_j = [a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j}]^{\mathrm{T}}$ are respectively an eigenvalue and an eigenvector of the system

$$\left[\mathbf{Q} + p_j\left(\mathbf{R} + \mathbf{R}^{\mathrm{T}}\right) + p_j^2\mathbf{T}\right]\mathbf{A}_j = 0 \tag{7}$$

with the elements of the 5×5 matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{T}$ defined as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{\mathrm{E}} & \mathbf{e}_{11} & \mathbf{h}_{11} \\ \mathbf{e}_{11}^{\mathrm{T}} & -\alpha_{11} & -d_{11} \\ \mathbf{h}_{11}^{\mathrm{T}} & -d_{11} & -\mu_{11} \end{bmatrix}, \;\; \mathbf{R} = \begin{bmatrix} \mathbf{R}^{\mathrm{E}} & \mathbf{e}_{31} & \mathbf{h}_{31} \\ \mathbf{e}_{13}^{\mathrm{T}} & -\alpha_{13} & -d_{13} \\ \mathbf{h}_{13}^{\mathrm{T}} & -d_{13} & -\mu_{13} \end{bmatrix},$$
$$\mathbf{T} = \begin{bmatrix} \mathbf{R}^{\mathrm{E}} & \mathbf{e}_{33} & \mathbf{h}_{33} \\ \mathbf{e}_{33}^{\mathrm{T}} & -\alpha_{33} & -d_{33} \\ \mathbf{h}_{33}^{\mathrm{T}} & -d_{33} & -\mu_{33} \end{bmatrix}, \tag{8}$$

and

$$\left(\mathbf{Q}^{\mathrm{E}}\right)_{jk} = c_{1jk1}, \;\; \left(\mathbf{R}^{\mathrm{E}}\right)_{jk} = c_{1jk3}, \;\; \left(\mathbf{T}^{\mathrm{E}}\right)_{jk} = c_{3jk3},$$
$$(\mathbf{e}_{ij})_m = e_{ijm}, \;\; (\mathbf{h}_{ij})_m = h_{ijm} \tag{9}$$

The 5×5 matrix $\mathbf{B}$ can be found by the formulas

$$\mathbf{B} = \mathbf{R}^{\mathrm{T}}\mathbf{A} + \mathbf{TAP} \tag{10}$$

with $\mathbf{P} = \mathrm{diag}[p_1, p_2, p_3, p_4, p_5]$. The prime () denotes differentiation with respect to the argument, the overbar stands for the complex conjugate.

For transversely isotropic magnetoelectroelastic materials poled in the direction x_3 which have an essential practical significance, all the fields are independent of the coordinate x_2, the displacement V_2 of the vector function $\mathbf{V}$ decouples in the (x_1, x_3)-plane from the components (V_1, V_3, V_4, V_5). In the following chapters, our attention will be focused on the plane problem for the components (V_1, V_3, V_4, V_5).

3. Statement of the problem and solutions

3.1. A magnetoelectroelastic bimaterial plane with an interface crack

A bimaterial composed of two dissimilar magnetoelectroelastic semi-infinite planes $x_3 > 0$ and $x_3 < 0$ with material properties defined by the following material constants $c_{ijks}^{(1)}$, $e_{iks}^{(1)}$, $h_{iks}^{(1)}$, $d_{is}^{(1)}$, $\alpha_{is}^{(1)}$, $\mu_{si}^{(1)}$ and $c_{ijks}^{(2)}$, $e_{iks}^{(2)}$, $h_{iks}^{(2)}$, $d_{is}^{(2)}$, $\alpha_{is}^{(2)}$, $\mu_{si}^{(2)}$, respectively, is considered (Fig. 1). We assume, that the vector $\mathbf{t}$ is continuous across the whole bimaterial interface, that the part $L = (-\infty, c) \cup (b, \infty)$ of the